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Expected Time to Recruitment in A Two - Grade Manpower System Using Order Statistics for Inter-Decision Times and Attrition

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ABSTRACT
In this paper, a two-grade organization subjected to random exit of personal due to policy decisions taken by the organization is considered. There is an associated loss of manpower if a person quits. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds for each grade-one is optional and the other mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach three mathematical models are constructed using an appropriate univariate policy of recruitment. Performance measures namely mean and variance of the time to recruitment are obtained for all the models when (i) the loss of man-hours and the inter decision time forms an order statistics (ii) the optional and mandatory thresholds follow different distributions. The analytical results are substantiated by numerical illustrations and the influence of nodal parameters on the performance measures is also analyzed.
Keywords-Manpower planning, Shock models, Univariate recruitment policy, Mean and variance of the time to recruitment.

I. INTRODUCTION

Consider an organization having two grades in which depletion of manpower occurs at every decision epoch. In the univariate policy of recruitment based on shock model approach, recruitment is made as and when the cumulative loss of manpower crosses a threshold. Employing this recruitment policy, expected time to recruitment is obtained under different conditions for several models in [1], [2] and [3].Recently in [4],for a single grade man-power system with a mandatory exponential threshold for the loss of manpower ,the authors have obtained the system performance measures namely mean and variance of the time to recruitment when the inter-decision times form an order statistics .In [2] , for a single grade manpower system ,the author has considered a new recruitment policy involving two thresholds for the loss of man-power in the organization in which one is optional and the other is mandatory and obtained the mean time to recruitment under different conditions on the nature of thresholds. In [5-8] the authors have extended the results in [2] for a two-grade system according as the thresholds are exponential random variables or geometric random variables or SCBZ property possessing random variables or extended exponential random variables. In [9-17], the authors have extended the results in [4] for a two-grade system involving two thresholds by assuming different distributions for thresholds under different condition of inter-decision time and wastage. The objective of the present paper is to obtain the performance measures when (i) the loss of man-hours and the inter decision time forms an order statistics (ii) the optional and mandatory thresholds follow different distributions.

II. MODEL DESCRIPTION AND ANALYSIS OF MODEL –I

Consider an organization taking decisions at random epoch in (0, ∞) and at every decision epoch a random number of persons quit the organization. There is an associated loss of man-hours if a person quits. It is assumed that the loss of man-hours are linear and cumulative. Let X_i be the loss of man-hours due to the i^{th} decision epoch , i=1,2,3…n Let $X_{(1)}, X_{(2)}, X_{(3)}, \ldots X_{(n)}$ be the order statistics selected from the sample $X_1, X_2, \ldots X_n$ with respective density functions $g_{X(1)}(.), g_{X(2)}(.), \ldots g_{X(n)}(.)$. Let U_i be a continuous random variable denoting inter-decision time between $(i-1)^{th}$ and i^{th} decision, i=1,2, 3….k with cumulative distribution function F(.), probability density function f(.) and mean $\frac{1}{\lambda}$ (λ>0). Let $U_{(1)}$ ($U_{(k)}$) be the smallest (largest) order statistic with probability density function. $f_{u(1)}(.),(f_{u(k)}(.))$.Let Y_1, Y_2 (Z_1, Z_2) denotes the optional (mandatory) thresholds for the loss of man-hours in grades 1 and 2, with parameters $\theta_1, \theta_2, \alpha_1, \alpha_2$ respectively, where $\theta_1, \theta_2, \alpha_1, \alpha_2$ are positive. It is assumed that $Y_1 < Z_1$ and $Y_2 < Z_2$. Write Y=Max (Y_1, Y_2) and Z=Max (Z_1, Z_2), where Y (Z) is the optional (mandatory) threshold for the loss of man-hours in the organization. The loss of man- hours, optional and mandatory thresholds are assumed as statistically independent. Let T be the time to recruitment in the organization with cumulative distribution function L (.), probability density function l (.), mean E (T) and variance V (T). Let $F_k(.)$ be the k fold convolution of F(.). Let $l^*(.)$ and $f^*(.)$, be the Laplace transform of l (.) and f (.), respectively. Let $V_k(t)$ be the probability that there are exactly k decision epochs in (0, t]. It is known from Renewal theory [18] that $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hours crosses optional threshold Y. The Univariate recruitment policy employed in this paper is as follows: If the total loss of man-hours exceeds the optional threshold Y, the organization may or may not go for recruitment. But if the total loss of man-hours exceeds the mandatory threshold Z, the recruitment is necessary.

MAIN RESULTS

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i \le Y\right) + p \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i > Y\right) \times P\left(\sum_{i=1}^{k} X_i < Z\right) \quad (1)$$

For n=1, 2, 3…k the probability density function of $X_{(n)}$ is given by

$$g_{x(n)}(t) = n\, kc_n\, [G(t)]^{n-1} g(t)[1-G(t)]^{k-n}, n = 1,2,3..k \quad (2)$$

If $g(t) = g_{x(1)}(t)$

In this case it is known that

$$g_{x(1)}(t) = k\, g(t)(1-G(t))^{k-1} \quad (3)$$

By hypothesis $g(t) = ce^{-ct}$ (4)

Therefore from (3) and (4) we get,

$$g^*_{x(1)}(\theta) = \frac{kc}{kc+\theta} \quad (5)$$

If $g(t) = g_{x(k)}(t)$

In this case it is known that

$$g_{x(k)}(t) = k(G(t))^{k-1} g(t) \quad (6)$$

Therefore from (4) and (6) we get

$$g^*_{x(k)}(\theta) = \frac{k!c^k}{(\theta+c)(\theta+2c)...(\theta+kc)} \quad (7)$$

For r=1, 2, 3…k the probability density function of $U_{(r)}$ is given by

$$f_{u(r)}(t) = r\, kc_r\, [F(t)]^{r-1} f(t)[1-F(t)]^{k-r}, r = 1,2,3..k \quad (8)$$

If $f(t) = f_{u(1)}(t)$

In this case it is known that

$$f_{u(1)}(t) = k\, f(t)(1-F(t))^{k-1} \quad (9)$$

Therefore from (8) and (9) we get,

$$f^*_{u(1)}(s) = \frac{k\lambda}{k\lambda+s} \quad (10)$$

If $f(t) = f_{u(k)}(t)$

In this case it is known that

$$f_{u(k)}(t) = k(F(t))^{k-1} f(t) \quad (11)$$

$$f^*_{u(k)}(s) = \frac{k!\lambda^k}{(s+\lambda)(s+2\lambda)...(s+k\lambda)} \quad (12)$$

It is known that

$$E(T) = -\left.\frac{d(l^*(s))}{ds}\right|_{s=0}, E(T^2) = \left.\frac{d^2(l^*(s))}{ds^2}\right|_{s=0} \text{ and } V(T) = E(T^2) - (E(T))^2 \quad (13)$$

C**ase (i)**: The distribution of optional and mandatory thresholds follow exponential distribution

For this case the first two moments of time to recruitment are found to be

If $g(t) = g_{x(1)}(t), f(t) = f_{u(1)}(t)$

$$E(T) = C_{I1} + C_{I2} - C_{I3} + p(C_{I4} + C_{I5} - C_{I6} - H_{I1,4} - H_{I1,5} + H_{I1,6} - H_{I2,4} - H_{I2,5} + H_{I2,6} + H_{I3,4} + H_{I3,5} - H_{I3,6}) \quad (14)$$

$$E(T^2) = 2\left(C_{I1}^2 + C_{I2}^2 - C_{I3}^2 + p\left(C_{I4}^2 + C_{I5}^2 - C_{I6}^2 - H_{I1,4}^2 - H_{I1,5}^2 + H_{I1,6}^2 - H_{I2,4}^2 - H_{I2,5}^2 + H_{I2,6}^2 + H_{I3,4}^2 + H_{I3,5}^2 - H_{I3,6}^2\right)\right) \quad (15)$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6.

$$C_{Ia} = \frac{1}{k\lambda(1 - D_{Ia})} \text{ and } H_{Ib,d} = \frac{1}{k\lambda(1 - D_{Ib} D_{Id})} \quad (16)$$

$D_{I1} = g^*_{x(1)}(\theta_1), D_{I2} = g^*_{x(1)}(\theta_2), D_{I3} = g^*_{x(1)}(\theta_1 + \theta_2), D_{I4} = g^*_{x(1)}(\alpha_1), D_{I5} = g^*_{x(1)}(\alpha_2), D_{I6} = g^*_{x(1)}(\alpha_1 + \alpha_2)$

are given by (5) (17)

If $g(t) = g_{x(k)}(t), f(t) = f_{u(1)}(t)$

$$E(T) = L_{K1} + L_{K2} - L_{K3} + p(L_{K4} + L_{K5} - L_{K6} - M_{K1,4} - M_{K1,5} + M_{K1,6} - M_{K2,4} - M_{K2,5} + M_{K2,6} + M_{K3,4} + M_{K3,5} - M_{K3,6}) \quad (18)$$

$$E(T^2) = 2\left(L_{K1}^2 + L_{K2}^2 - L_{K3}^2 + p\left(L_{K4}^2 + L_{K5}^2 - L_{K6}^2 - M_{K1,4}^2 - M_{K1,5}^2 + M_{K1,6}^2 - M_{K2,4}^2 - M_{K2,5}^2 + M_{K2,6}^2 + M_{K3,4}^2 + M_{K3,5}^2 - M_{K3,6}^2\right)\right) \quad (19)$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6.

$$L_{Ka} = \frac{1}{k\lambda(1 - D_{Ka})} \text{ and } M_{Kb,d} = \frac{1}{k\lambda(1 - D_{Kb} D_{Kd})} \quad (20)$$

$D_{K1} = g^*_{x(k)}(\theta_1), D_{K2} = g^*_{x(k)}(\theta_2), D_{K3} = g^*_{x(k)}(\theta_1 + \theta_2), D_{K4} = g^*_{x(k)}(\alpha_1), D_{K5} = g^*_{x(k)}(\alpha_2), D_{K6} = g^*_{x(k)}(\alpha_1 + \alpha_2)$

are given by (7) (21)

If $g(t) = g_{x(k)}(t), f(t) = f_{u(k)}(t)$

$$E(T) = P_{K1} + P_{K2} - P_{K3} + p(P_{K4} + P_{K5} - P_{K6} - Q_{K1,4} - Q_{K1,5} + Q_{K1,6} - Q_{K2,4} - Q_{K2,5} + Q_{K2,6} + Q_{K3,4} + Q_{K3,5} - Q_{K3,6}) \quad (22)$$

$$E(T^2) = 2\left(P_{K1}^2 + P_{K2}^2 - P_{K3}^2 + p\left(P_{K4}^2 + P_{K5}^2 - P_{K6}^2 - Q_{K1,4}^2 - Q_{K1,5}^2 + Q_{K1,6}^2 - Q_{K2,4}^2 - Q_{K2,5}^2 + Q_{K2,6}^2 + Q_{K3,4}^2 + Q_{K3,5}^2 - Q_{K3,6}^2\right)\right) - \frac{1}{\lambda^2}\left(N^2 - M\right)\left(\frac{1}{1-D_{K1}} + \frac{1}{1-D_{K2}} - \frac{1}{1-D_{K3}}\right)$$
$$- \frac{p}{\lambda^2}\left(N^2 - M\right)\left(\frac{1}{1-D_{K4}} + \frac{1}{1-D_{K5}} - \frac{1}{1-D_{K6}} - \frac{1}{1-D_{K1}D_{K4}} - \frac{1}{1-D_{K1}D_{K5}} + \frac{1}{1-D_{K1}D_{K6}} - \frac{1}{1-D_{K2}D_{K4}} - \frac{1}{1-D_{K2}D_{K5}} + \frac{1}{1-D_{K2}D_{K6}} + \frac{1}{1-D_{K3}D_{K4}} + \frac{1}{1-D_{K3}D_{K5}} - \frac{1}{1-D_{K3}D_{K6}}\right) \quad (23)$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6.

$$P_{Ka} = \frac{N}{\lambda(1 - D_{Ka})}, Q_{Kb,d} = \frac{N}{\lambda(1 - D_{Kb} D_{Kd})}, N = \sum_{n=1}^{k} \frac{1}{n} \text{ and } M = \sum_{n=1}^{k} \frac{1}{n^2} \quad (24)$$

If $g(t) = g_{x(1)}(t), f(t) = f_{u(k)}(t)$

$$E(T) = R_{I1} + R_{I2} - R_{I3} + p(R_{I4} + R_{I5} - R_{I6} - S_{I1,4} - S_{I1,5} + S_{I1,6} - S_{I2,4} - S_{I2,5} + S_{I2,6} + S_{I3,4} + S_{I3,5} - S_{I3,6}) \quad (25)$$

$$E(T^2) = 2\left(R_{I1}^2 + R_{I2}^2 - R_{I3}^2 + p\left(R_{I4}^2 + R_{I5}^2 - R_{I6}^2 - S_{I1,4}^2 - S_{I1,5}^2 + S_{I1,6}^2 - S_{I2,4}^2 S_{I2,5}^2 + S_{I2,6}^2 + S_{I3,4}^2 + S_{I3,5}^2 - S_{I3,6}^2\right)\right) - \frac{1}{\lambda^2}\left(N^2 - M\right)\left(\frac{1}{1-D_{I1}} + \frac{1}{1-D_{I2}} - \frac{1}{1-D_{I3}}\right)$$
$$- \frac{p}{\lambda^2}\left(N^2 - M\right)\left(\frac{1}{1-D_{I4}} + \frac{1}{1-D_{I5}} - \frac{1}{1-D_{I6}} - \frac{1}{1-D_{I1}D_{I4}} - \frac{1}{1-D_{I1}D_{I5}} + \frac{1}{1-D_{I1}D_{I6}} - \frac{1}{1-D_{I2}D_{I4}} - \frac{1}{1-D_{I2}D_{I5}} + \frac{1}{1-D_{I2}D_{I6}} + \frac{1}{1-D_{I3}D_{I4}} + \frac{1}{1-D_{I3}D_{I5}} - \frac{1}{1-D_{I3}D_{I6}}\right) \quad (26)$$

where for a = 1, 2…6, b=1, 2, 3 and d=4, 5

$$R_{Ia} = \frac{N}{\lambda(1 - D_{Ia})}, S_{Ib,d} = \frac{N}{\lambda(1 - D_{Ib} D_{Id})}, N = \sum_{n=1}^{k} \frac{1}{n} \text{ and } M = \sum_{n=1}^{k} \frac{1}{n^2} \quad (27)$$

Case (ii): The distributions of optional and mandatory thresholds follow extended exponential distribution with shape parameter 2.

If $g(t) = g_{x(1)}(t), f(t) = f_{u(1)}(t)$

$$E(T) = 2C_{I1} + 2C_{I2} + 2C_{I7} + 2C_{I8} - C_{I9} - C_{I10} - C_{I11} - 4C_{I3} + p(2C_{I4} + 2C_{I5} + 2C_{I12} + 2C_{I13} - C_{I14} - C_{I15} - C_{I16} - 4C_{I6} - 4H_{I1,4} - 4H_{I1,5} - 4H_{I1,12} - 4H_{I1,13}$$
$$+ 2H_{I1,14} + 2H_{I1,15} + 2H_{I1,16} + 8H_{I1,6} - 4H_{I2,4} - 4H_{I2,5} - 4H_{I2,12} - 4H_{I2,13} + 2H_{I2,14} + 2H_{I2,15} + 2H_{I2,16} + 8H_{I2,6} - 4H_{I7,4} - 4H_{I7,5} - 4H_{I7,12} - 4H_{I7,13}$$
$$+ 2H_{I7,14} + 2H_{I7,15} + 2H_{I7,16} + 8H_{I7,6} - 4H_{I8,4} - 4H_{I8,5} - 4H_{I8,12} - 4H_{I8,13} + 2H_{I8,14} + 2H_{I8,15} + 2H_{I8,16} + 8H_{I8,6} + 2H_{I9,4} + 2H_{I9,5} + 2H_{I9,12} + 2H_{I9,13}$$
$$- H_{I9,14} - H_{I9,15} - H_{I9,16} - 4H_{I9,6} + 2H_{I10,4} + 2H_{I10,5} + 2H_{I10,12} + 2H_{I10,13} - H_{I10,14} - H_{I10,15} - H_{I10,16} - 4H_{I10,6} + 2H_{I11,4} + 2H_{I11,5} + 2H_{I11,12} + 2H_{I11,13}$$
$$- H_{I11,14} - H_{I11,15} - H_{I11,16} - 4H_{I11,6} + 8H_{I3,4} + 8H_{I3,5} + 8H_{I3,12} + 8H_{I3,13} - 4H_{I3,14} - 4H_{I3,15} - 4H_{I3,16} - 16H_{I3,6}) \quad (28)$$

$$E(T^2) = 2\left[2C_{I1}^2 + 2C_{I2}^2 + 2C_{I7}^2 + 2C_{I8}^2 - C_{I9}^2 - C_{I10}^2 - C_{I11}^2 - 4C_{I3}^2 + p\left(2C_{I4}^2 + 2C_{I5}^2 + 2C_{I12}^2 + 2C_{I13}^2 - C_{I14}^2 - C_{I15}^2 - C_{I16}^2 - 4C_{I6}^2 - 4H_{I1,4}^2 - 4H_{I1,5}^2 - 4H_{I1,12}^2\right.\right.$$
$$- 4H_{I1,13}^2 + 2H_{I1,14}^2 + 2H_{I1,15}^2 + 2H_{I1,16}^2 + 8H_{I1,6}^2 - 4H_{I2,4}^2 - 4H_{I2,5}^2 - 4H_{I2,12}^2 - 4H_{I2,13}^2 + 2H_{I2,14}^2 + 2H_{I2,15}^2 + 2H_{I2,16}^2 + 8H_{I2,6}^2 - 4H_{I7,4}^2 - 4H_{I7,5}^2$$
$$- 4H_{I7,12}^2 - 4H_{I7,13}^2 + 2H_{I7,14}^2 + 2H_{I7,15}^2 + 2H_{I7,16}^2 + 8H_{I7,6}^2 - 4H_{I8,4}^2 - 4H_{I8,5}^2 - 4H_{I8,12}^2 - 4H_{I8,13}^2 + 2H_{I8,14}^2 + 2H_{I8,15}^2 + 2H_{I8,16}^2 + 8H_{I8,6}^2 + 2H_{I9,4}^2$$
$$+ 2H_{I9,5}^2 + 2H_{I9,12}^2 + 2H_{I9,13}^2 - H_{I9,14}^2 - H_{I9,15}^2 - H_{I9,16}^2 - 4H_{I9,6}^2 + 2H_{I10,4}^2 + 2H_{I10,5}^2 + 2H_{I10,12}^2 + 2H_{I10,13}^2 - H_{I10,14}^2 - H_{I10,15}^2 - H_{I10,16}^2 - 4H_{I10,6}^2 + 2H_{I11,4}^2$$
$$\left.+ 2H_{I11,5}^2 + 2H_{I11,12}^2 + 2H_{I11,13}^2 - H_{I11,14}^2 - H_{I11,15}^2 - H_{I11,16}^2 - 4H_{I11,6}^2 + 8H_{I3,4}^2 + 8H_{I3,5}^2 + 8H_{I3,12}^2 + 8H_{I3,13}^2 - 4H_{I3,14}^2 - 4H_{I3,15}^2 - 4H_{I3,16}^2 - 16H_{I3,6}^2\right) \quad (29)$$

where for a=1, 2, 3…16, b=1, 2, 3,7, 8, 9, 10,11 and d=4, 5, 6, 12, 13,14,15,16.
$C_{Ia}, H_{Ib,d}$ are given by (16)

$$D_{I7}=g^*_{x(1)}(2\theta_1+\theta_2), D_{I8}=g^*_{x(1)}(\theta_1+2\theta_2), D_{I9}=g^*_{x(1)}(2\theta_1), D_{I10}=g^*_{x(1)}(2\theta_2)\ D_{I11}=g^*_{x(1)}(2\theta_1+2\theta_2), \quad (30)$$

$$D_{I12}=g^*_{x(1)}(2\alpha_1+\alpha_2), D_{I13}=g^*_{x(1)}(2\alpha_2+\alpha_1), D_{I14}=g^*_{x(1)}(2\alpha_1), D_{I15}=g^*_{x(1)}(2\alpha_2), D_{I16}=g^*_{x(1)}(2\alpha_1+2\alpha_2)$$

If $g(t)=g_{x(k)}(t), f(t)=f_{u(1)}(t)$

$$\begin{aligned}E(T)=\ & 2L_{K1}+2L_{K2}+2L_{K7}+2L_{K8}-L_{KI9}-L_{K10}-L_{K11}-4L_{K3}+p(2L_{K4}+2L_{K5}+2L_{K12}+2L_{K13}-L_{K14}-L_{K15}-L_{K16}-4L_{K6}-4M_{K1,4}-4M_{K1,5}-4M_{K1,12}-4M_{K1,13}\\ &+2M_{K1,14}+2M_{K1,15}+2M_{K1,16}+8M_{K1,6}-4M_{K2,4}-4M_{K2,5}-4M_{K2,12}-4M_{K2,13}+2M_{K2,14}+2M_{K2,15}+2M_{K2,16}+8M_{K2,6}-4M_{K7,4}-4M_{K7,5}-4M_{K7,12}-4M_{K7,13}\\ &+2M_{K7,14}+2M_{K7,15}+2M_{K7,16}+8M_{K7,6}-4M_{K8,4}-4M_{K8,5}-4M_{K8,12}-4M_{K8,13}+2M_{K8,14}+2M_{K8,15}+2M_{K8,16}+8M_{K8,6}+2M_{K9,4}+2M_{K9,5}+2M_{K9,12}+2M_{K9,13}\\ &-M_{K9,14}-M_{K9,15}-M_{K9,16}-4M_{K9,6}+2M_{K10,4}+2M_{K10,5}+2M_{K10,12}+2M_{K10,13}-M_{K10,14}-M_{I10,15}-M_{K10,16}-4M_{K10,6}+2M_{K11,4}+2M_{K11,5}+2M_{K11,12}+2M_{K11,13}\\ &-M_{K11,14}-M_{K11,15}-M_{K11,16}-4M_{K11,6}+8M_{K3,4}+8M_{K3,5}+8M_{K3,12}+8M_{K3,13}-4M_{K3,14}-4M_{K3,15}-4M_{K3,16}-16M_{K3,6})\end{aligned} \quad (31)$$

$$\begin{aligned}E(T^2)=2\big[\ & 2L^2_{K1}+2L^2_{K2}+2L^2_{K7}+2L^2_{K8}-L^2_{K9}-L^2_{K10}-L^2_{K11}-4L^2_{K3}+p(2L^2_{K4}+2L^2_{K5}+2L^2_{K12}+2L^2_{K13}-L^2_{K14}-L^2_{K15}-L^2_{K16}-4L^2_{K6}\\ &-4M^2_{K1,4}-4M^2_{K1,5}-4M^2_{K1,12}-4M^2_{K1,13}+2M^2_{K1,14}+2M^2_{K1,15}+2M^2_{K1,16}+8M^2_{K1,6}-4M^2_{K2,4}-4M^2_{K2,5}-4M^2_{K2,12}-4M^2_{K2,13}+2M^2_{K2,14}\\ &+2M^2_{K2,15}+2M^2_{K2,16}+8M^2_{K2,6}-4M^2_{K7,4}-4M^2_{K7,5}-4M^2_{K7,12}-4M^2_{K7,13}+2M^2_{K7,14}+2M^2_{K7,15}+2M^2_{K7,16}+8M^2_{K7,6}-4M^2_{K8,4}-4M^2_{K8,5}\\ &-4M^2_{K8,12}-4M^2_{K8,13}+2M^2_{K8,14}+2M^2_{K8,15}+2M^2_{K8,16}+8M^2_{K8,6}+2M^2_{K9,4}+2M^2_{K9,5}+2M^2_{K9,12}+2M^2_{K9,13}-M^2_{K9,14}-M^2_{K9,15}-M^2_{K9,16}\\ &-4M^2_{K9,6}+2M^2_{K10,4}+2M^2_{K10,5}+2M^2_{K10,12}+2M^2_{K10,13}-M^2_{K10,14}-M^2_{K10,15}-M^2_{K10,16}-4M^2_{K10,6}+2M^2_{K11,4}+2M^2_{K11,5}+2M^2_{K11,12}+2M^2_{K11,13}\\ &-M^2_{K11,14}-M^2_{K11,15}-M^2_{K11,16}-4M^2_{K11,6}+8M^2_{K3,4}+8M^2_{K3,5}+8M^2_{K3,12}+8M^2_{K3,13}-4M^2_{K3,14}-4M^2_{K3,15}-4M^2_{K3,16}-16M^2_{K3,6})\end{aligned} \quad (32)$$

where for a=1,2,3,4,5,….16,b=1,2,…8 and d=9,10…..16. $L_{Ka}, M_{Kb,d}$ are given by (20) .

If $g(t)=g_{x(k)}(t), f(t)=f_{u(1)}(t)$

$$\begin{aligned}E(T)=\ & 2P_{K1}+2P_{K2}+2P_{K7}+2P_{K8}-P_{K9}-P_{K10}-P_{K11}-4P_{K3}+p(2P_{K4}+2P_{K5}+2P_{K12}+2P_{K13}-P_{K14}-P_{K15}-P_{K16}-4P_{K6}-4Q_{K1,4}-4Q_{K1,5}-4Q_{K1,12}\\ &-4Q_{K1,13}+2Q_{K1,14}+2Q_{K1,15}+2Q_{K1,16}+8Q_{K1,6}-4Q_{K2,4}-4Q_{K2,5}-4Q_{K2,12}-4Q_{K2,13}+2Q_{K2,14}+2Q_{K2,15}+2Q_{K2,16}+8Q_{K2,6}-4Q_{K7,4}-4Q_{K7,5}\\ &-4Q_{K7,12}-4Q_{K7,13}+2Q_{K7,14}+2Q_{K7,15}+2Q_{K7,16}+8Q_{K7,6}-4Q_{K8,4}-4Q_{K8,5}-4Q_{K8,12}-4Q_{K8,13}+2Q_{K8,14}+2Q_{K8,15}+2Q_{K8,16}+8Q_{K8,6}+2Q_{K9,4}\\ &+2Q_{K9,5}+2Q_{K9,12}+2Q_{K9,13}-Q_{K9,14}-Q_{K9,15}-Q_{K9,16}-4Q_{K9,6}+2Q_{K10,4}+2Q_{K10,5}+2Q_{K10,12}+2Q_{K10,13}-Q_{K10,14}-Q_{K10,15}-Q_{K10,16}-4Q_{K10,6}+2Q_{K11,4}\\ &+2Q_{K11,5}+2Q_{K11,12}+2Q_{K11,13}-Q_{K11,14}-Q_{K11,15}-Q_{K11,16}-4Q_{K11,6}+8Q_{K3,4}+8Q_{K3,5}+8Q_{K3,12}+8Q_{K3,13}-4Q_{K3,14}-4Q_{K3,15}-4Q_{K3,16}-16Q_{K3,6})\end{aligned} \quad (33)$$

$$\begin{aligned}E(T^2)=2\Big[\ & 2P^2_{K1}+2P^2_{K2}+2P^2_{K7}+2P^2_{K8}-P^2_{K9}-P^2_{K10}-P^2_{K11}-4P^2_{K3}+p(2P^2_{K4}+2P^2_{K5}+2P^2_{K12}+2P^2_{K13}-P^2_{K14}-P^2_{K15}-P^2_{K16}-4P^2_{K6}-4Q^2_{K1,4}-4Q^2_{K1,5}-4Q^2_{K1,12}-4Q^2_{K1,13}\\ &+2Q^2_{K1,14}+2Q^2_{K1,15}+2Q^2_{K1,16}+8Q^2_{K1,6}-4Q^2_{K2,4}-4Q^2_{K2,5}-4Q^2_{K2,12}-4Q^2_{K2,13}+2Q^2_{K2,14}+2Q^2_{K2,15}+2Q^2_{K2,16}+8Q^2_{K2,6}-4Q^2_{K7,4}-4Q^2_{K7,5}-4Q^2_{K7,12}-4Q^2_{K7,13}\\ &+2Q^2_{K7,14}+2Q^2_{K7,15}+2Q^2_{K7,16}+8Q^2_{K7,6}-4Q^2_{K8,4}-4Q^2_{K8,5}-4Q^2_{K8,12}-4Q^2_{K8,13}+2Q^2_{K8,14}+2Q^2_{K8,15}+2Q^2_{K8,16}+8Q^2_{K8,6}+2Q^2_{K9,4}+2Q^2_{K9,5}+2Q^2_{K9,12}+2Q^2_{K9,13}\\ &-Q^2_{K9,14}-Q^2_{K9,15}-Q^2_{K9,16}-4Q^2_{K9,6}+2Q^2_{K10,4}+2Q^2_{K10,5}+2Q^2_{K10,12}+2Q^2_{K10,13}-Q^2_{K10,14}-Q^2_{K10,15}-Q^2_{K10,16}-4Q^2_{K10,6}+2Q^2_{K11,4}+2Q^2_{K11,5}+2Q^2_{K11,12}+2Q^2_{K11,13}\\ &-Q^2_{K11,14}-Q^2_{K11,15}-Q^2_{K11,16}-4Q^2_{K11,6}+8Q^2_{K3,4}+8Q^2_{K3,5}+8Q^2_{K3,12}+8Q^2_{K3,13}-4Q^2_{K3,14}-4Q^2_{K3,15}-4Q^2_{K3,16}-16Q^2_{K3,6})\Big]-\frac{1}{\lambda^2}(N^2-M)\Big(\frac{2}{1-D_{K1}}+\frac{2}{1-D_{K2}}+\frac{2}{1-D_{K7}}\\ &+\frac{2}{1-D_{K8}}-\frac{1}{1-D_{K9}}-\frac{1}{1-D_{K10}}-\frac{1}{1-D_{K11}}-\frac{4}{1-D_{K3}}\Big)-\frac{p}{\lambda^2}(N^2-M)\Big(\frac{2}{1-D_{K4}}+\frac{2}{1-D_{K5}}+\frac{2}{1-D_{K12}}+\frac{2}{1-D_{K13}}-\frac{1}{1-D_{K14}}-\frac{1}{1-D_{K15}}-\frac{1}{1-D_{K16}}-\frac{4}{1-D_{K6}}-\frac{4}{1-D_{K1}D_{K4}}\\ &-\frac{4}{1-D_{K1}D_{K5}}-\frac{4}{1-D_{K1}D_{K12}}-\frac{4}{1-D_{K1}D_{K13}}+\frac{2}{1-D_{K1}D_{K14}}+\frac{2}{1-D_{K1}D_{K15}}+\frac{2}{1-D_{K1}D_{K16}}+\frac{8}{1-D_{K1}D_{K6}}-\frac{4}{1-D_{K2}D_{K4}}-\frac{4}{1-D_{K2}D_{K5}}-\frac{4}{1-D_{K2}D_{K12}}-\frac{4}{1-D_{K2}D_{K13}}\\ &+\frac{2}{1-D_{K2}D_{K14}}+\frac{2}{1-D_{K2}D_{K15}}+\frac{2}{1-D_{K2}D_{K16}}+\frac{8}{1-D_{K2}D_{K6}}-\frac{4}{1-D_{K7}D_{K4}}-\frac{4}{1-D_{K7}D_{K5}}-\frac{4}{1-D_{K7}D_{K12}}-\frac{4}{1-D_{K7}D_{K13}}+\frac{2}{1-D_{K7}D_{K14}}+\frac{2}{1-D_{K7}D_{K15}}+\frac{2}{1-D_{K7}D_{K16}}\\ &+\frac{8}{1-D_{K7}D_{K6}}-\frac{4}{1-D_{K8}D_{K4}}-\frac{4}{1-D_{K8}D_{K5}}-\frac{4}{1-D_{K8}D_{K12}}-\frac{4}{1-D_{K8}D_{K13}}+\frac{2}{1-D_{K8}D_{K14}}+\frac{2}{1-D_{K8}D_{K15}}+\frac{2}{1-D_{K8}D_{K16}}+\frac{8}{1-D_{K8}D_{K6}}+\frac{2}{1-D_{K9}D_{K4}}+\frac{2}{1-D_{K9}D_{K5}}\\ &+\frac{2}{1-D_{K9}D_{K12}}+\frac{2}{1-D_{K9}D_{K13}}-\frac{1}{1-D_{K9}D_{K14}}-\frac{1}{1-D_{K9}D_{K15}}-\frac{1}{1-D_{K9}D_{K16}}-\frac{4}{1-D_{K9}D_{K6}}+\frac{2}{1-D_{K10}D_{K4}}+\frac{2}{1-D_{K10}D_{K5}}+\frac{2}{1-D_{K10}D_{K12}}+\frac{2}{1-D_{K10}D_{K13}}-\frac{1}{1-D_{K10}D_{K14}}\\ &-\frac{1}{1-D_{K10}D_{K15}}-\frac{1}{1-D_{K10}D_{K16}}-\frac{4}{1-D_{K10}D_{K6}}+\frac{2}{1-D_{K11}D_{K4}}+\frac{2}{1-D_{K11}D_{K5}}+\frac{2}{1-D_{K11}D_{K12}}+\frac{2}{1-D_{K11}D_{K13}}-\frac{1}{1-D_{K11}D_{K14}}-\frac{1}{1-D_{K11}D_{K15}}-\frac{1}{1-D_{K11}D_{K16}}-\frac{4}{1-D_{K11}D_{K6}}\\ &+\frac{8}{1-D_{K3}D_{K4}}+\frac{8}{1-D_{K3}D_{K5}}+\frac{8}{1-D_{K3}D_{K12}}+\frac{8}{1-D_{K3}D_{K13}}-\frac{4}{1-D_{K3}D_{K14}}-\frac{4}{1-D_{K3}D_{K15}}-\frac{4}{1-D_{K3}D_{K16}}-\frac{16}{1-D_{K3}D_{K6}}\Big)\end{aligned} \quad (34)$$

where for a=1,2,3,4,5,….16,b=1,2,…8 and d=9,10…..16. $P_{Ka}, Q_{Kb,d}$ are given by (24) .

If $g(t)=g_{x(1)}(t), f(t)=f_{u(k)}(t)$

$$\begin{aligned}E(T)=\ & 2R_{I1}+2R_{I2}+2R_{I7}+2R_{I8}-R_{I9}-R_{I10}-R_{I11}-4R_{I3}+p(2R_{I4}+2R_{I5}+2R_{I12}+2R_{I13}-R_{I14}-R_{I15}-R_{I16}-4R_{I6}-4S_{I1,4}-4S_{I1,5}-4S_{I1,12}\\ &-4S_{I1,13}+2S_{I1,14}+2S_{I1,15}+2S_{I1,16}+8S_{I1,6}-4S_{I2,4}-4S_{I2,5}-4S_{I2,12}-4S_{I2,13}+2S_{I2,14}+2S_{I2,15}+2S_{I2,16}+8S_{I2,6}-4S_{I7,4}-4S_{I7,5}-4S_{I7,12}\\ &-4S_{I7,13}+2S_{I7,14}+2S_{I7,15}+2S_{I7,16}+8S_{I7,6}-4S_{I8,4}-4S_{I8,5}-4S_{I8,12}-4S_{I8,13}+2S_{I8,14}+2S_{I8,15}+2S_{I8,16}+8S_{I8,6}+2S_{I9,4}+2S_{I9,5}+2S_{I9,12}\\ &+2S_{I9,13}-S_{I9,14}-S_{I9,15}-S_{I9,16}-4S_{I9,6}+2S_{I10,4}+2S_{I10,5}+2S_{I10,12}+2S_{I10,13}-S_{I10,14}-S_{I10,15}-S_{I10,16}-4S_{I10,6}+2S_{I11,4}+2S_{I11,5}+2S_{I11,12}\\ &+2S_{I11,13}-S_{I11,14}-S_{I11,15}-S_{I11,16}-4S_{I11,6}+8S_{I3,4}+8S_{I3,5}+8S_{I3,12}+8S_{I3,13}-4S_{I3,14}-4S_{I3,15}-4S_{I3,16}-16S_{I3,6})\end{aligned} \quad (35)$$

$$
\begin{aligned}
E(T^2) = 2\Big[& 2R_{I1}^2+2R_{I2}^2+2R_{I7}^2+2R_{I8}^2-R_{I9}^2-R_{I10}^2-R_{I11}^2-4R_{I3}^2+p\big(2R_{I4}^2+2R_{I5}^2+2R_{I12}^2+2R_{I13}^2-R_{I14}^2-R_{I15}^2-R_{I16}^2-4R_{I6}^2-4S_{I1,4}^2-4S_{I1,5}^2-4S_{I1,12}^2-4S_{I1,13}^2 \\
&+2S_{I1,14}^2+2S_{I1,15}^2+2S_{I1,16}^2+8S_{I1,6}^2-4S_{I2,4}^2-4S_{I2,5}^2-4S_{I2,12}^2-4S_{I2,13}^2+2S_{I2,14}^2+2S_{I2,15}^2+2S_{I2,16}^2+8S_{I2,6}^2-4S_{I7,4}^2-4S_{I7,5}^2-4S_{I7,12}^2-4S_{I7,13}^2+2S_{I7,14}^2+2S_{I7,15}^2 \\
&+2S_{I7,16}^2+8S_{I7,6}^2-4S_{I8,4}^2-4S_{I8,5}^2-4S_{I8,12}^2-4S_{I8,13}^2+2S_{I8,14}^2+2S_{I8,15}^2+2S_{I8,16}^2+8S_{I8,6}^2+2S_{I9,4}^2+2S_{I9,5}^2+2S_{I9,12}^2+2S_{I9,13}^2-S_{I9,14}^2-S_{I9,15}^2-S_{I9,16}^2-4S_{I9,6}^2+2S_{I10,4}^2 \\
&+2S_{I10,5}^2+2S_{I10,12}^2+2S_{I10,13}^2-S_{I10,14}^2-S_{I10,15}^2-S_{I10,16}^2-4S_{I10,6}^2+2S_{I11,4}^2+2S_{I11,5}^2+2S_{I11,12}^2+2S_{I11,13}^2-S_{I11,14}^2-S_{I11,15}^2-S_{I11,16}^2-4S_{I11,6}^2+8S_{I3,4}^2+8S_{I3,5}^2 \\
&+8S_{I3,12}^2+8S_{I3,13}^2-4S_{I3,14}^2-4S_{I3,15}^2-4S_{I3,16}^2-16S_{I3,6}^2\Big]-\frac{1}{\lambda^2}(N^2-M)\left(\frac{2}{1-D_{I1}}+\frac{2}{1-D_{I2}}+\frac{2}{1-D_{I7}}+\frac{2}{1-D_{I8}}-\frac{1}{1-D_{I9}}-\frac{1}{1-D_{I10}}-\frac{1}{1-D_{I11}}-\frac{4}{1-D_{I3}}\right) \\
&-\frac{p}{\lambda^2}(N^2-M)\left(\frac{2}{1-D_{I4}}+\frac{2}{1-D_{I5}}+\frac{2}{1-D_{I12}}+\frac{2}{1-D_{I13}}-\frac{1}{1-D_{I14}}-\frac{1}{1-D_{I15}}-\frac{1}{1-D_{I16}}-\frac{4}{1-D_{I6}}-\frac{4}{1-D_{I1}D_{I4}}-\frac{4}{1-D_{I1}D_{I5}}-\frac{4}{1-D_{I1}D_{I12}}-\frac{4}{1-D_{I1}D_{I13}}\right. \\
&+\frac{2}{1-D_{I1}D_{I14}}+\frac{2}{1-D_{I1}D_{I15}}+\frac{2}{1-D_{I1}D_{I16}}+\frac{8}{1-D_{I1}D_{I6}}-\frac{4}{1-D_{I2}D_{I4}}-\frac{4}{1-D_{I2}D_{I5}}-\frac{4}{1-D_{I2}D_{I12}}-\frac{4}{1-D_{I2}D_{I13}}+\frac{2}{1-D_{I2}D_{I14}}+\frac{2}{1-D_{I2}D_{I15}}+\frac{2}{1-D_{I2}D_{I16}} \\
&+\frac{8}{1-D_{I2}D_{I6}}-\frac{4}{1-D_{I7}D_{I4}}-\frac{4}{1-D_{I7}D_{I5}}-\frac{4}{1-D_{I7}D_{I12}}-\frac{4}{1-D_{I7}D_{I13}}+\frac{2}{1-D_{I7}D_{I14}}+\frac{2}{1-D_{I7}D_{I15}}+\frac{2}{1-D_{I7}D_{I16}}+\frac{8}{1-D_{I7}D_{I6}}-\frac{4}{1-D_{I8}D_{I4}}-\frac{4}{1-D_{I8}D_{I5}} \\
&-\frac{4}{1-D_{I8}D_{I12}}-\frac{4}{1-D_{I8}D_{I13}}+\frac{2}{1-D_{I8}D_{I14}}+\frac{2}{1-D_{I8}D_{I15}}+\frac{2}{1-D_{I8}D_{I16}}+\frac{8}{1-D_{I8}D_{I6}}+\frac{2}{1-D_{I9}D_{I4}}+\frac{2}{1-D_{I9}D_{I5}}+\frac{2}{1-D_{I9}D_{I12}}+\frac{2}{1-D_{I9}D_{I13}}-\frac{1}{1-D_{I9}D_{I14}} \\
&-\frac{1}{1-D_{I9}D_{I15}}-\frac{1}{1-D_{I9}D_{I16}}-\frac{4}{1-D_{I9}D_{I6}}+\frac{2}{1-D_{I10}D_{I4}}+\frac{2}{1-D_{I10}D_{I5}}+\frac{2}{1-D_{I10}D_{I12}}+\frac{2}{1-D_{I10}D_{I13}}-\frac{1}{1-D_{I10}D_{I14}}-\frac{1}{1-D_{I10}D_{I15}}-\frac{1}{1-D_{I10}D_{I16}}-\frac{4}{1-D_{I10}D_{I6}} \\
&+\frac{2}{1-D_{I11}D_{I4}}+\frac{2}{1-D_{I11}D_{I5}}+\frac{2}{1-D_{I11}D_{I12}}+\frac{2}{1-D_{I11}D_{I13}}-\frac{1}{1-D_{I11}D_{I14}}-\frac{1}{1-D_{I11}D_{I15}}-\frac{1}{1-D_{I11}D_{I16}}-\frac{4}{1-D_{I11}D_{I6}}+\frac{8}{1-D_{I3}D_{I4}}+\frac{8}{1-D_{I3}D_{I5}}+\frac{8}{1-D_{I3}D_{I12}} \\
&\left.+\frac{8}{1-D_{I3}D_{I13}}-\frac{4}{1-D_{I3}D_{I14}}-\frac{4}{1-D_{I3}D_{I15}}-\frac{4}{1-D_{I3}D_{I16}}-\frac{16}{1-D_{I3}D_{I6}}\right)
\end{aligned}
\tag{36}
$$

where for a=1,2,3…16, b=1, 2, 3, 7, 8, 9, 10,11 and d=4, 5, 6, 12, 13,14,15,16.
$R_{Ia}, S_{Ib,d}$ are given by (27).

Case (iii): The distributions of optional and mandatory thresholds possess SCBZ property.
If $g(t)=g_{x(1)}(t), f(t)=f_{u(1)}(t)$

$$
\begin{aligned}
E(T) = {} & p_2C_{I1}+q_2C_{I2}+p_1C_{I3}-p_1p_2C_{I4}-p_1q_2C_{I5}+q_1C_{I6}-p_2q_1C_{I7}-q_1q_2C_{I8}+p(p_4C_{I9}+q_4C_{I10}+p_3C_{I11}-p_3p_4C_{I12}-p_3q_4C_{I13}+q_3C_{I14} \\
&-p_4q_3C_{I15}-q_3q_4C_{I16}-p_2p_4H_{I1,9}-p_2q_4H_{I1,10}-p_2p_3H_{I1,11}+p_2p_3p_4H_{I1,12}+p_2p_3q_4H_{I1,13}-p_2q_3H_{I1,14}+p_2p_4q_3H_{I1,15}+p_2q_3q_4H_{I1,16} \\
&-q_2p_4H_{I2,9}-q_2q_4H_{I2,10}-q_2p_3H_{I2,11}+q_2p_3p_4H_{I2,12}+q_2p_3q_4H_{I2,13}-q_2q_3H_{I2,14}+q_2p_4q_3H_{I2,15}+q_2q_3q_4H_{I2,16}-p_1p_4H_{I3,9} \\
&-p_1q_4H_{I3,10}-p_1p_3H_{I3,11}+p_1p_3p_4H_{I3,12}+p_1p_3q_4H_{I3,13}-p_1q_3H_{I3,14}+p_1p_4q_3H_{I3,15}+p_1q_3q_4H_{I3,16}+p_1p_2p_4H_{I4,9}+p_1p_2q_4H_{I4,10} \\
&+p_1p_2p_3H_{I4,11}-p_1p_2p_3p_4H_{I4,12}-p_1p_2p_3q_4H_{I4,13}+p_1p_2q_3H_{I4,14}-p_1p_2p_4q_3H_{I4,15}-p_1p_2q_3q_4H_{I4,16}+p_1q_2p_4H_{I5,9}+p_1q_2q_4H_{I5,10} \\
&+p_1q_2p_3H_{I5,11}-p_1q_2p_3p_4H_{I5,12}-p_1q_2p_3q_4H_{I5,13}+p_1q_2q_3H_{I5,14}-p_1q_2p_4q_3H_{I5,15}-p_1q_2q_3q_4H_{I5,16}-q_1p_4H_{I6,9}-q_1q_4H_{I6,10}-q_1p_3H_{I6,11} \\
&+q_1p_3p_4H_{I6,12}+q_1p_3q_4H_{I6,13}-q_1q_3H_{I6,14}+q_1p_4q_3H_{I6,15}+q_1q_3q_4H_{I6,16}+q_1p_2p_4H_{I7,9}+q_1p_2q_4H_{I7,10}+q_1p_2p_3H_{I7,11} \\
&-q_1p_2p_3p_4H_{I7,12}-q_1p_2p_3q_4H_{I7,13}+q_1p_2q_3H_{I7,14}-q_1p_2p_4q_3H_{I7,15}-q_1p_2q_3q_4H_{I7,16}+q_1q_2p_4H_{I8,9}+q_1q_2q_4H_{I8,10} \\
&+q_1q_2p_3H_{I8,11}-q_1q_2p_3p_4H_{I8,12}-q_1q_2p_3q_4H_{I8,13}+q_1q_2q_3H_{I8,14}-q_1q_2p_4q_3H_{I8,15}-q_1q_2q_3q_4H_{I8,16})
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
E(T^2) = 2\Big(& p_2C_{I1}^2+q_2C_{I2}^2+p_1C_{I3}^2-p_1p_2C_{I4}^2-p_1q_2C_{I5}^2+q_1C_{I6}^2-p_2q_1C_{I7}^2-q_1q_2C_{I8}^2+p\big(p_4C_{I9}^2+q_4C_{I10}^2+p_3C_{I11}^2-p_3p_4C_{I12}^2-p_3q_4C_{I13}^2+q_3C_{I14}^2 \\
&-p_4q_3C_{I15}^2-q_3q_4C_{I16}^2-p_2p_4H_{I1,9}^2-p_2q_4H_{I1,10}^2-p_2p_3H_{I1,11}^2+p_2p_3p_4H_{I1,12}^2+p_2p_3q_4H_{I1,13}^2-p_2q_3H_{I1,14}^2+p_2p_4q_3H_{I1,15}^2+p_2q_3q_4H_{I1,16}^2 \\
&-q_2p_4H_{I2,9}^2-q_2q_4H_{I2,10}^2-q_2p_3H_{I2,11}^2+q_2p_3p_4H_{I2,12}^2+q_2p_3q_4H_{I2,13}^2-q_2q_3H_{I2,14}^2+q_2p_4q_3H_{I2,15}^2+q_2q_3q_4H_{I2,16}^2-p_1p_4H_{I3,9}^2-p_1q_4H_{I3,10}^2 \\
&-p_1p_3H_{I3,11}^2+p_1p_3p_4H_{I3,12}^2+p_1p_3q_4H_{I3,13}^2-p_1q_3H_{I3,14}^2+p_1p_4q_3H_{I3,15}^2+p_1q_3q_4H_{I3,16}^2+p_1p_2p_4H_{I4,9}^2+p_1p_2q_4H_{I4,10}^2+p_1p_2p_3H_{I4,11}^2 \\
&-p_1p_2p_3p_4H_{I4,12}^2-p_1p_2p_3q_4H_{I4,13}^2+p_1p_2q_3H_{I4,14}^2-p_1p_2p_4q_3H_{I4,15}^2-p_1p_2q_3q_4H_{I4,16}^2+p_1q_2p_4H_{I5,9}^2+p_1q_2q_4H_{I5,10}^2+p_1q_2p_3H_{I5,11}^2 \\
&-p_1q_2p_3p_4H_{I5,12}^2-p_1q_2p_3q_4H_{I5,13}^2+p_1q_2q_3H_{I5,14}^2-p_1q_2p_4q_3H_{I5,15}^2-p_1q_2q_3q_4H_{I5,16}^2-q_1p_4H_{I6,9}^2-q_1q_4H_{I6,10}^2-q_1p_3H_{I6,11}^2+q_1p_3p_4H_{I6,12}^2 \\
&+q_1p_3q_4H_{I6,13}^2-q_1q_3H_{I6,14}^2+q_1p_4q_3H_{I6,15}^2+q_1q_3q_4H_{I6,16}^2+q_1p_2p_4H_{I7,9}^2+q_1p_2q_4H_{I7,10}^2+q_1p_2p_3H_{I7,11}^2-q_1p_2p_3p_4H_{I7,12}^2-q_1p_2p_3q_4H_{I7,13}^2 \\
&+q_1p_2q_3H_{I7,14}^2-q_1p_2p_4q_3H_{I7,15}^2-q_1p_2q_3q_4H_{I7,16}^2+q_1q_2p_4H_{I8,9}^2+q_1q_2q_4H_{I8,10}^2+q_1q_2p_3H_{I8,11}^2-q_1q_2p_3p_4H_{I8,12}^2-q_1q_2p_3q_4H_{I8,13}^2+q_1q_2q_3H_{I8,14}^2 \\
&-q_1q_2p_4q_3H_{I8,15}^2-q_1q_2q_3q_4H_{I8,16}^2\big)\Big)
\end{aligned}
\tag{38}
$$

where for a=1,2….16, b=1,2,3,4,5,6,7,8 and d=9,10,11,12,13,14,15,16.

$$
C_{Ia}=\frac{1}{k\lambda(1-B_{Ia})}\text{ and }H_{Ib,d}=\frac{1}{k\lambda(1-B_{Ib}B_{Id})} \tag{39}
$$

$$
p_1=\frac{(\delta_1-\eta_1)}{(\mu_1+\delta_1-\eta_1)},\ p_2=\frac{(\delta_2-\eta_2)}{(\mu_2+\delta_2-\eta_2)},\ p_3=\frac{(\delta_3-\eta_3)}{(\mu_3+\delta_3-\eta_3)},\ p_4=\frac{(\delta_4-\eta_4)}{(\mu_4+\delta_4-\eta_4)}
$$

$$
q_1=1-p_1,\ q_2=1-p_2,\ q_3=1-p_3,\ q_4=1-p_4 \tag{40}
$$

$$
\begin{aligned}
&B_{I1}=g_{x(1)}^*(\delta_2+\mu_2),\ B_{I2}=g_{x(1)}^*(\eta_2),\ B_{I3}=g_{x(1)}^*(\delta_1+\mu_1),\ B_{I4}=g_{x(1)}^*(\delta_1+\mu_1+\delta_2+\mu_2), \\
&B_{I5}=g_{x(1)}^*(\delta_1+\eta_2+\mu_2),\ B_{I6}=g_{x(1)}^*(\eta_1),\ B_{I7}=g_{x(1)}^*(\eta_1+\delta_2+\mu_2),\ B_{I8}=g_{x(1)}^*(\eta_1+\eta_2), \\
&B_{I9}=g_{x(1)}^*(\delta_4+\mu_4),\ B_{I10}=g_{x(1)}^*(\eta_4),\ B_{I11}=g_{x(1)}^*(\delta_3+\mu_3),\ B_{I12}=g_{x(1)}^*(\delta_3+\mu_3+\delta_4+\mu_4), \\
&B_{I13}=g_{x(1)}^*(\delta_3+\eta_4+\mu_3),\ B_{I14}=g_{x(1)}^*(\eta_3),\ B_{I15}=g_{x(1)}^*(\eta_3+\delta_4+\mu_4),\ B_{I16}=g_{x(1)}^*(\eta_3+\eta_4)
\end{aligned}
\tag{41}
$$

If $g(t) = g_{x(k)}(t), f(t) = f_{u(1)}(t)$

$$
\begin{aligned}
E(T) = {} & p_2 L_{K1} + q_2 L_{K2} + p_1 L_{K3} - p_1 p_2 L_{K4} - p_1 q_2 L_{K5} + q_1 L_{K6} - p_2 q_1 L_{K7} - q_1 q_2 L_{K8} + p(p_4 L_{K9} + q_4 L_{K10} + p_3 L_{K11} - p_3 p_4 L_{K12} - p_3 q_4 L_{K13} + q_3 L_{K14} - p_4 q_3 L_{K15} \\
& - q_3 q_4 L_{K16} - p_2 p_4 M_{K1,9} - p_2 q_4 M_{K1,10} - p_2 p_3 M_{K11,11} + p_2 p_3 p_4 M_{K1,12} + p_2 p_3 q_4 M_{K1,13} - p_2 q_3 M_{K1,14} + p_2 p_4 q_3 M_{K1,15} + p_2 q_3 q_4 M_{K1,16} - q_2 p_4 M_{K2,9} \\
& - q_2 q_4 M_{K2,10} - q_2 p_3 M_{K2,11} + q_2 p_3 p_4 M_{K2,12} + q_2 p_3 q_4 M_{K2,13} - q_2 q_3 M_{K2,14} + q_2 p_4 q_3 M_{K2,15} + q_2 q_3 q_4 M_{K2,16} - p_1 p_4 M_{K3,9} - p_1 q_4 M_{K3,10} - p_1 p_3 M_{K3,11} \\
& + p_1 p_3 p_4 M_{K3,12} + p_1 p_3 q_4 M_{K3,13} - p_1 q_3 M_{I3,14} + p_1 p_4 q_3 M_{I3,15} + p_1 q_3 q_4 M_{I3,16} + p_1 p_2 p_4 M_{I4,9} + p_1 p_2 q_4 M_{K4,10} + p_1 p_2 p_3 M_{K4,11} - p_1 p_2 p_3 p_4 M_{K4,12} \\
& - p_1 p_2 p_3 q_4 M_{I4,13} + p_1 p_2 q_3 M_{I4,14} - p_1 p_2 p_4 q_3 M_{I4,15} - p_1 p_2 q_3 q_4 M_{I4,16} + p_1 q_2 p_4 M_{K5,9} + p_1 q_2 q_4 M_{K5,10} + p_1 q_2 p_3 M_{K5,11} - p_1 q_2 p_3 p_4 M_{K5,12} - p_1 q_2 p_3 q_4 M_{K5,13} \\
& + p_1 q_2 q_3 M_{K5,14} - p_1 q_2 p_4 q_3 M_{K5,15} - p_1 q_2 q_3 q_4 M_{K5,16} - q_1 p_4 M_{K6,9} - q_1 q_4 M_{K6,10} - q_1 p_3 M_{K6,11} + q_1 p_3 p_4 M_{K6,12} + q_1 p_3 q_4 M_{K6,13} - q_1 q_3 M_{K6,14} + q_1 p_4 q_3 M_{K6,15} \\
& + q_1 q_3 q_4 M_{K6,16} + q_1 p_2 p_4 M_{K7,9} + q_1 p_2 q_4 M_{K7,10} + q_1 p_2 p_3 M_{K7,11} - q_1 p_2 p_3 p_4 M_{K7,12} - q_1 p_2 p_3 q_4 M_{K7,13} + q_1 p_2 q_3 M_{K7,14} - q_1 p_2 p_4 q_3 M_{K7,15} - q_1 p_2 q_3 q_4 M_{K7,16} \\
& + q_1 q_2 p_4 M_{K8,9} + q_1 q_2 q_4 M_{K8,10} + q_1 q_2 p_3 M_{K8,11} - q_1 q_2 p_3 p_4 M_{K8,12} - q_1 q_2 p_3 q_4 M_{K8,13} + q_1 q_2 q_3 M_{K8,14} - q_1 q_2 p_4 q_3 M_{K8,15} - q_1 q_2 q_3 q_4 M_{K8,16})
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
E(T^2) = 2\big(& p_2 L_{K1}^2 + q_2 L_{K2}^2 + p_1 L_{K3}^2 - p_1 p_2 L_{K4}^2 - p_1 q_2 L_{K5}^2 + q_1 L_{K6}^2 - p_2 q_1 L_{K7}^2 - q_1 q_2 L_{K8}^2 + p\big(p_4 L_{K9}^2 + q_4 L_{K10}^2 + p_3 L_{K11}^2 - p_3 p_4 L_{K12}^2 - p_3 q_4 L_{K13}^2 + q_3 L_{K14}^2 \\
& - p_4 q_3 L_{K15}^2 - q_3 q_4 L_{K16}^2 - p_2 p_4 M_{K1,9}^2 - p_2 q_4 M_{K1,10}^2 - p_2 p_3 M_{K1,11}^2 + p_2 p_3 p_4 M_{K1,12}^2 + p_2 p_3 q_4 M_{K1,13}^2 - p_2 q_3 M_{K1,14}^2 + p_2 p_4 q_3 M_{K1,15}^2 + p_2 q_3 q_4 M_{K1,16}^2 \\
& - q_2 p_4 M_{K2,9}^2 - q_2 q_4 M_{K2,10}^2 - q_2 p_3 M_{K2,11}^2 + q_2 p_3 p_4 M_{K2,12}^2 + q_2 p_3 q_4 M_{K2,13}^2 - q_2 q_3 M_{K2,14}^2 + q_2 p_4 q_3 M_{K2,15}^2 + q_2 q_3 q_4 M_{K2,16}^2 - p_1 p_4 M_{K3,9}^2 - p_1 q_4 M_{K3,10}^2 \\
& - p_1 p_3 M_{K3,11}^2 + p_1 p_3 p_4 M_{K3,12}^2 + p_1 p_3 q_4 M_{K3,13}^2 - p_1 q_3 M_{K3,14}^2 + p_1 p_4 q_3 M_{K3,15}^2 + p_1 q_3 q_4 M_{K3,16}^2 + p_1 p_2 p_4 M_{K4,9}^2 + p_1 p_2 q_4 M_{K4,10}^2 + p_1 p_2 p_3 M_{K4,11}^2 \\
& - p_1 p_2 p_3 p_4 M_{K4,12}^2 - p_1 p_2 p_3 q_4 M_{K4,13}^2 + p_1 p_2 q_3 M_{K4,14}^2 - p_1 p_2 p_4 q_3 M_{K4,15}^2 - p_1 p_2 q_3 q_4 M_{K4,16}^2 + p_1 q_2 p_4 M_{K5,9}^2 + p_1 q_2 q_4 M_{K5,10}^2 + p_1 q_2 p_3 M_{K5,11}^2 - p_1 q_2 p_3 p_4 M_{K5,12}^2 \\
& - p_1 q_2 p_3 q_4 M_{K5,13}^2 + p_1 q_2 q_3 M_{K5,14}^2 - p_1 q_2 p_4 q_3 M_{K5,15}^2 - p_1 q_2 q_3 q_4 M_{K5,16}^2 - q_1 p_4 M_{K6,9}^2 - q_1 q_4 M_{K6,10}^2 - q_1 p_3 M_{K6,11}^2 + q_1 p_3 p_4 M_{K6,12}^2 + q_1 p_3 q_4 M_{K6,13}^2 - q_1 q_3 M_{K6,14}^2 \\
& + q_1 p_4 q_3 M_{K6,15}^2 + q_1 q_3 q_4 M_{K6,16}^2 + q_1 p_2 p_4 M_{K7,9}^2 + q_1 p_2 q_4 M_{K7,10}^2 + q_1 p_2 p_3 M_{K7,11}^2 - q_1 p_2 p_3 p_4 M_{K7,12}^2 - q_1 p_2 p_3 q_4 M_{K7,13}^2 + q_1 p_2 q_3 M_{K7,14}^2 - q_1 p_2 p_4 q_3 M_{K7,15}^2 \\
& - q_1 p_2 q_3 q_4 M_{K7,16}^2 + q_1 q_2 p_4 M_{K8,9}^2 + q_1 q_2 q_4 M_{K8,10}^2 + q_1 q_2 p_3 M_{K8,11}^2 - q_1 q_2 p_3 p_4 M_{K8,12}^2 - q_1 q_2 p_3 q_4 M_{K8,13}^2 + q_1 q_2 q_3 M_{K8,14}^2 - q_1 q_2 p_4 q_3 M_{K8,15}^2 - q_1 q_2 q_3 q_4 M_{K8,16}^2\big)\big)
\end{aligned}
\tag{43}
$$

where for a=1,2….16, b=1,2,3,4,5,6,7,8 and d=9,10,11,12,13,14,15,16.

$$
L_{Ka} = \frac{1}{k\lambda(1 - B_{Ka})} \text{ and } M_{Kb,d} = \frac{1}{k\lambda(1 - B_{Kb} B_{Kd})}
\tag{44}
$$

$$
\begin{aligned}
& B_{K1} = g^*_{x(k)}(\delta_2 + \mu_2), B_{K2} = g^*_{x(k)}(\eta_2), B_{K3} = g^*_{x(k)}(\delta_1 + \mu_1), B_{K4} = g^*_{x(k)}(\delta_1 + \mu_1 + \delta_2 + \mu_2), \\
& B_{K5} = g^*_{x(k)}(\delta_1 + \eta_2 + \mu_2), B_{K6} = g^*_{x(k)}(\eta_1), B_{K7} = g^*_{x(k)}(\eta_1 + \delta_2 + \mu_2), B_{K8} = g^*_{x(k)}(\eta_1 + \eta_2), \\
& B_{K9} = g^*_{x(k)}(\delta_4 + \mu_4), B_{K10} = g^*_{x(k)}(\eta_4), B_{K11} = g^*_{x(k)}(\delta_3 + \mu_3), B_{K12} = g^*_{x(k)}(\delta_3 + \mu_3 + \delta_4 + \mu_4), \\
& B_{K13} = g^*_{x(k)}(\delta_3 + \eta_4 + \mu_3), B_{K14} = g^*_{x(k)}(\eta_3), B_{K15} = g^*_{x(k)}(\eta_3 + \delta_4 + \mu_4), B_{K16} = g^*_{x(k)}(\eta_3 + \eta_4)
\end{aligned}
\tag{45}
$$

If $g(t) = g_{x(k)}(t), f(t) = f_{u(k)}(t)$

$$
\begin{aligned}
E(T) = {} & p_2 P_{K1} + q_2 P_{K2} + p_1 P_{K3} - p_1 p_2 P_{K4} - p_1 q_2 P_{K5} + q_1 P_{K6} - p_2 q_1 P_{K7} - q_1 q_2 P_{K8} + p(p_4 P_{K9} + q_4 P_{K10} + p_3 P_{K11} - p_3 p_4 P_{K12} \\
& - p_3 q_4 P_{K13} + q_3 P_{K14} - p_4 q_3 P_{K15} - q_3 q_4 P_{K16} - p_2 p_4 Q_{K1,9} - p_2 q_4 Q_{K1,10} - p_2 p_3 Q_{K1,11} + p_2 p_3 p_4 Q_{K1,12} + p_2 p_3 q_4 Q_{K1,13} \\
& - p_2 q_3 Q_{K1,14} + p_2 p_4 q_3 Q_{K1,15} + p_2 q_3 q_4 Q_{K1,16} - q_2 p_4 Q_{K2,9} - q_2 q_4 Q_{K2,10} - q_2 p_3 Q_{K2,11} + q_2 p_3 p_4 Q_{K2,12} + q_2 p_3 q_4 Q_{K2,13} \\
& - q_2 q_3 Q_{K2,14} + q_2 p_4 q_3 Q_{K2,15} + q_2 q_3 q_4 Q_{K2,16} - p_1 p_4 Q_{K3,9} - p_1 q_4 Q_{K3,10} - p_1 p_3 Q_{K3,11} + p_1 p_3 p_4 Q_{K3,12} + p_1 p_3 q_4 Q_{K3,13} \\
& - p_1 q_3 Q_{K3,14} + p_1 p_4 q_3 Q_{K3,15} + p_1 q_3 q_4 Q_{K3,16} + p_1 p_2 p_4 Q_{K4,9} + p_1 p_2 q_4 Q_{K4,10} + p_1 p_2 p_3 Q_{K4,11} - p_1 p_2 p_3 p_4 Q_{K4,12} \\
& - p_1 p_2 p_3 q_4 Q_{K4,13} + p_1 p_2 q_3 Q_{K4,14} - p_1 p_2 p_4 q_3 Q_{K4,15} - p_1 p_2 q_3 q_4 Q_{K4,16} + p_1 q_2 p_4 Q_{K5,9} + p_1 q_2 q_4 Q_{K5,10} + p_1 q_2 p_3 Q_{K5,11} \\
& - p_1 q_2 p_3 p_4 Q_{K5,12} - p_1 q_2 p_3 q_4 Q_{K5,13} + p_1 q_2 q_3 Q_{K5,14} - p_1 q_2 p_4 q_3 Q_{K5,15} - p_1 q_2 q_3 q_4 Q_{K5,16} - q_1 p_4 Q_{K6,9} - q_1 q_4 Q_{K6,10} \\
& - q_1 p_3 Q_{K6,11} + q_1 p_3 p_4 Q_{K6,12} + q_1 p_3 q_4 Q_{K6,13} - q_1 q_3 Q_{K6,14} + q_1 p_4 q_3 Q_{K6,15} + q_1 q_3 q_4 Q_{K6,16} + q_1 p_2 p_4 Q_{K7,9} + q_1 p_2 q_4 Q_{K7,10} \\
& + q_1 p_2 p_3 Q_{K7,11} - q_1 p_2 p_3 p_4 Q_{K7,12} - q_1 p_2 p_3 q_4 Q_{K7,13} + q_1 p_2 q_3 Q_{K7,14} - q_1 p_2 p_4 q_3 Q_{K7,15} - q_1 p_2 q_3 q_4 Q_{K7,16} + q_1 q_2 p_4 Q_{K8,9} \\
& + q_1 q_2 q_4 Q_{K8,10} + q_1 q_2 p_3 Q_{K8,11} - q_1 q_2 p_3 p_4 Q_{K8,12} - q_1 q_2 p_3 q_4 Q_{K8,13} + q_1 q_2 q_3 Q_{K8,14} - q_1 q_2 p_4 q_3 Q_{K8,15} - q_1 q_2 q_3 q_4 Q_{K8,16})
\end{aligned}
\tag{46}
$$

$$
\begin{aligned}
E(T^2) = 2\Big(& p_2P^2_{K1} + q_2P^2_{K2} + p_1P^2_{K3} - p_1p_2P^2_{K4} - p_1q_2P^2_{K5} + q_1P^2_{K6} - p_2q_1P^2_{K7} - q_1q_2P^2_{K8} + p\big(p_4P^2_{K9} + q_4P^2_{K10} + p_3P^2_{K11} - p_3p_4P^2_{K12} \\
& - p_3q_4P^2_{K13} + q_3P^2_{K14} + p_4q_3P^2_{K15} - q_3q_4P^2_{K16} - p_2p_4Q^2_{K1,9} - p_2q_4Q^2_{K1,10} - p_2p_3Q^2_{K1,11} + p_2p_3p_4Q^2_{K1,12} \\
& + p_2p_3q_4Q^2_{K1,13} - p_2q_3Q^2_{K1,14} + p_2p_4q_3Q^2_{K1,15} + p_2q_3q_4Q^2_{K1,16} - q_2p_4Q^2_{K2,9} - q_2q_4Q^2_{K2,10} - q_2p_3Q^2_{K2,11} \\
& + q_2p_3p_4Q^2_{K2,12} + q_2p_3q_4Q^2_{K2,13} - q_2q_3Q^2_{K2,14} + q_2p_4q_3Q^2_{K2,15} + q_2q_3q_4Q^2_{K2,16} - p_1p_4Q^2_{K3,9} - p_1q_4Q^2_{K3,10} \\
& - p_1p_3Q^2_{K3,11} + p_1p_3p_4Q^2_{K3,12} + p_1p_3q_4Q^2_{K3,13} - p_1q_3Q^2_{K3,14} + p_1p_4q_3Q^2_{K3,15} + p_1q_3q_4Q^2_{K3,16} + p_1p_2p_4Q^2_{K4,9} \\
& + p_1p_2q_4Q^2_{K4,10} + p_1p_2p_3Q^2_{K4,11} - p_1p_2p_3p_4Q^2_{K4,12} - p_1p_2p_3q_4Q^2_{K4,13} + p_1p_2q_3Q^2_{K4,14} - p_1p_2p_4q_3Q^2_{K4,15} \\
& - p_1p_2q_3q_4Q^2_{K4,16} + p_1q_2p_4Q^2_{K5,9} + p_1q_2q_4Q^2_{K5,10} + p_1q_2p_3Q^2_{K5,11} - p_1q_2p_3p_4Q^2_{K5,12} - p_1q_2p_3q_4Q^2_{K5,13} \\
& + p_1q_2q_3Q^2_{K5,14} - p_1q_2p_4q_3Q^2_{K5,15} - p_1q_2q_3q_4Q^2_{K5,16} - q_1p_4Q^2_{K6,9} - q_1q_4Q^2_{K6,10} - q_1p_3Q^2_{K6,11} + q_1p_3p_4Q^2_{K6,12} \\
& + q_1p_3q_4Q^2_{K6,13} - q_1q_3Q^2_{K6,14} + q_1p_4q_3Q^2_{K6,15} + q_1q_3q_4Q^2_{K6,16} + q_1p_2p_4Q^2_{K7,9} + q_1p_2q_4Q^2_{K7,10} + q_1p_2p_3Q^2_{K7,11} \\
& - q_1p_2p_3p_4Q^2_{K7,12} - q_1p_2p_3q_4Q^2_{K7,13} + q_1p_2q_3Q^2_{K7,14} - q_1p_2p_4q_3Q^2_{K7,15} - q_1p_2q_3q_4Q^2_{K7,16} + q_1q_2p_4Q^2_{K8,9} \\
& + q_1q_2q_4Q^2_{K8,10} + q_1q_2p_3Q^2_{K8,11} - q_1q_2p_3p_4Q^2_{K8,12} - q_1q_2p_3q_4Q^2_{K8,13} + q_1q_2q_3Q^2_{K8,14} - q_1q_2p_4q_3Q^2_{K8,15} - q_1q_2q_3q_4Q^2_{K8,16}\big)\Big) \\
& - \frac{1}{\lambda^2}(N^2 - M)\left(\frac{p_2}{1-B_{K1}} + \frac{q_2}{1-B_{K2}} + \frac{p_1}{1-B_{K3}} - \frac{p_1p_2}{1-B_{K4}} - \frac{p_1q_2}{1-B_{K5}} + \frac{q_1}{1-B_{K6}} - \frac{q_1p_2}{1-B_{K7}} - \frac{q_1q_2}{1-B_{K8}}\right) - \frac{p}{\lambda^2}(N^2 - M)\left(\frac{p_4}{1-B_{K9}} + \frac{q_4}{1-B_{K10}}\right. \\
& + \frac{p_3}{1-B_{K11}} - \frac{p_3p_4}{1-B_{K12}} - \frac{p_3q_4}{1-B_{K13}} + \frac{q_3}{1-B_{K14}} + \frac{q_3p_4}{1-B_{K15}} - \frac{q_3q_4}{1-B_{K16}} - \frac{p_4p_2}{1-B_{K1}B_{K9}} - \frac{p_2q_4}{1-B_{K1}B_{K10}} - \frac{p_2p_3}{1-B_{K1}B_{K11}} \\
& + \frac{p_2p_3p_4}{1-B_{K1}B_{K12}} + \frac{p_2p_3q_4}{1-B_{K1}B_{K13}} - \frac{p_2q_3}{1-B_{K1}B_{K14}} + \frac{p_2q_3p_4}{1-B_{K1}B_{K15}} + \frac{p_2q_3q_4}{1-B_{K1}B_{K16}} - \frac{q_2p_4}{1-B_{K2}B_{K9}} - \frac{q_2q_4}{1-B_{K2}B_{K10}} - \frac{q_2p_3}{1-B_{K2}B_{K11}} \\
& + \frac{q_2p_3p_4}{1-B_{K2}B_{K12}} + \frac{q_2p_3q_4}{1-B_{K2}B_{K13}} - \frac{q_2q_3}{1-B_{K2}B_{K14}} + \frac{q_2q_3p_4}{1-B_{K2}B_{K15}} + \frac{q_2q_3q_4}{1-B_{K2}B_{K16}} - \frac{p_4p_1}{1-B_{K3}B_{K9}} - \frac{p_1q_4}{1-B_{K3}B_{K10}} - \frac{p_1p_3}{1-B_{K3}B_{K11}} \\
& + \frac{p_1p_3p_4}{1-B_{K3}B_{K12}} + \frac{p_1p_3q_4}{1-B_{K3}B_{K13}} - \frac{p_1q_3}{1-B_{K3}B_{K14}} + \frac{p_1q_3p_4}{1-B_{K3}B_{K15}} + \frac{p_1q_3q_4}{1-B_{K3}B_{K16}} + \frac{p_1p_4p_2}{1-B_{K4}B_{K9}} + \frac{p_1p_2q_4}{1-B_{K4}B_{K10}} + \frac{p_1p_2p_3}{1-B_{K4}B_{K11}} \\
& - \frac{p_1p_2p_3p_4}{1-B_{K4}B_{K12}} - \frac{p_1p_2p_3q_4}{1-B_{K4}B_{K13}} + \frac{p_1p_2q_3}{1-B_{K4}B_{K14}} - \frac{p_1p_2q_3p_4}{1-B_{K4}B_{K15}} - \frac{p_1p_2q_3q_4}{1-B_{K4}B_{K16}} + \frac{p_1p_4q_2}{1-B_{K5}B_{K9}} + \frac{p_1q_2q_4}{1-B_{K5}B_{K10}} + \frac{p_1q_2p_3}{1-B_{K5}B_{K11}} \\
& - \frac{p_1q_2p_3p_4}{1-B_{K5}B_{K12}} - \frac{p_1q_2p_3q_4}{1-B_{K5}B_{K13}} + \frac{p_1q_2q_3}{1-B_{K5}B_{K14}} - \frac{p_1q_2q_3p_4}{1-B_{K5}B_{K15}} - \frac{p_1q_2q_3q_4}{1-B_{K5}B_{K16}} - \frac{p_4q_1}{1-B_{K6}B_{K9}} - \frac{q_1q_4}{1-B_{K6}B_{K10}} - \frac{q_1p_3}{1-B_{K6}B_{K11}} \\
& + \frac{q_1p_3p_4}{1-B_{K6}B_{K12}} + \frac{q_1p_3q_4}{1-B_{K6}B_{K13}} - \frac{q_1q_3}{1-B_{K6}B_{K14}} + \frac{q_1q_3p_4}{1-B_{K6}B_{K15}} + \frac{q_1q_3q_4}{1-B_{K6}B_{K16}} + \frac{q_1p_4p_2}{1-B_{K7}B_{K9}} + \frac{q_1p_2q_4}{1-B_{K7}B_{K10}} + \frac{q_1p_2p_3}{1-B_{K7}B_{K11}} \\
& - \frac{q_1p_2p_3p_4}{1-B_{K7}B_{K12}} - \frac{q_1p_2p_3q_4}{1-B_{K7}B_{K13}} + \frac{q_1p_2q_3}{1-B_{K7}B_{K14}} - \frac{q_1p_2q_3p_4}{1-B_{K7}B_{K15}} - \frac{q_1p_2q_3q_4}{1-B_{K7}B_{K16}} + \frac{q_1p_4q_2}{1-B_{K8}B_{K9}} + \frac{q_1q_2q_4}{1-B_{K8}B_{K10}} + \frac{q_1q_2p_3}{1-B_{K8}B_{K11}} \\
& \left. - \frac{q_1q_2p_3p_4}{1-B_{K8}B_{K12}} - \frac{q_1q_2p_3q_4}{1-B_{K8}B_{K13}} + \frac{q_1q_2q_3}{1-B_{K8}B_{K14}} - \frac{q_1q_2q_3p_4}{1-B_{K8}B_{K15}} - \frac{q_1q_2q_3q_4}{1-B_{K8}B_{K16}}\right)
\end{aligned} \tag{47}
$$

where for a=1,2….16, b=1,2,3,4,5,6,7,8 and d=9,10,11,12,13,14,15,16.

$$
P_{Ka} = \frac{N}{\lambda(1 - B_{Ka})} \text{ and } Q_{Kb,d} = \frac{N}{\lambda(1 - B_{Kb}B_{Kd})} \tag{48}
$$

If $g(t) = g_{x(1)}(t), f(t) = f_{u(k)}(t)$

$$
\begin{aligned}
E(T) = & p_2R_{I1} + q_2R_{I2} + p_1R_{I3} - p_1p_2R_{I4} - p_1q_2R_{I5} + q_1R_{I6} - p_2q_1R_{I7} - q_1q_2R_{I8} + p(p_4R_{I9} + q_4R_{I10} + p_3R_{I11} - p_3p_4R_{I12} - p_3q_4R_{I13} \\
& + q_3R_{I14} - p_4q_3R_{I15} - q_3q_4R_{I16} - p_2p_4S_{I1,9} - p_2q_4S_{I1,10} - p_2p_3S_{I1,11} + p_2p_3p_4S_{I1,12} + p_2p_3q_4S_{I1,13} - p_2q_3S_{I1,14} + p_2p_4q_3S_{I1,15} \\
& + p_2q_3q_4S_{I1,16} - q_2p_4S_{I2,9} - q_2q_4S_{I2,10} - q_2p_3S_{I2,11} + q_2p_3p_4S_{I2,12} + q_2p_3q_4S_{I2,13} - q_2q_3S_{I2,14} + q_2p_4q_3S_{I2,15} + q_2q_3q_4S_{I2,16} \\
& - p_1p_4S_{I3,9} - p_1q_4S_{I3,10} - p_1p_3S_{I3,11} + p_1p_3p_4S_{I3,12} + p_1p_3q_4S_{I3,13} - p_1q_3S_{I3,14} + p_1p_4q_3S_{I3,15} + p_1q_3q_4S_{I3,16} + p_1p_2p_4S_{I4,9} \\
& + p_1p_2q_4S_{I4,10} + p_1p_2p_3S_{I4,11} - p_1p_2p_3p_4S_{I4,12} - p_1p_2p_3q_4S_{I4,13} + p_1p_2q_3S_{I4,14} - p_1p_2p_4q_3S_{I4,15} - p_1p_2q_3q_4S_{I4,16} + p_1q_2p_4S_{I5,9} \\
& + p_1q_2q_4S_{I5,10} + p_1q_2p_3S_{I5,11} - p_1q_2p_3p_4S_{I5,12} - p_1q_2p_3q_4S_{I5,13} + p_1q_2q_3S_{I5,14} - p_1q_2p_4q_3S_{I5,15} - p_1q_2q_3q_4S_{I5,16} - q_1p_4S_{I6,9} \\
& - q_1q_4S_{I6,10} - q_1p_3S_{I6,11} + q_1p_3p_4S_{I6,12} + q_1p_3q_4S_{I6,13} - q_1q_3S_{I6,14} + q_1p_4q_3S_{I6,15} + q_1q_3q_4S_{I6,16} + q_1p_2p_4S_{I7,9} + q_1p_2q_4S_{I7,10} \\
& + q_1p_2p_3S_{I7,11} - q_1p_2p_3p_4S_{I7,12} - q_1p_2p_3q_4S_{I7,13} + q_1p_2q_3S_{I7,14} - q_1p_2p_4q_3S_{I7,15} - q_1p_2q_3q_4S_{I7,16} + q_1q_2p_4S_{I8,9} \\
& + q_1q_2q_4S_{I8,10} + q_1q_2p_3S_{I8,11} - q_1q_2p_3p_4S_{I8,12} - q_1q_2p_3q_4S_{I8,13} + q_1q_2q_3S_{I8,14} - q_1q_2p_4q_3S_{I8,15} - q_1q_2q_3q_4S_{I8,16})
\end{aligned} \tag{49}
$$

$$\begin{aligned}
E(T^2)=2\Big(&p_2R_{I1}^2+q_2R_{I2}^2+p_1R_{I3}^2-p_1p_2R_{I4}^2-p_1q_2R_{I5}^2+q_1R_{I6}^2-p_2q_1R_{I7}^2-q_1q_2R_{I8}^2+p\Big(p_4R_{I9}^2+q_4R_{I10}^2+p_3R_{I11}^2-p_3p_4R_{I12}^2-p_3q_4R_{I13}^2\\
&+q_3R_{I14}^2+p_4q_3R_{I15}^2-q_3q_4R_{I16}^2-p_2p_4S_{I1,9}^2-p_2q_4S_{I1,10}^2-p_2p_3S_{I1,11}^2+p_2p_3p_4S_{I1,12}^2+p_2p_3q_4S_{I1,13}^2-p_2q_3S_{I1,14}^2+p_2p_4q_3S_{I1,15}^2\\
&+p_2q_3q_4S_{I1,16}^2-q_2p_4S_{I2,9}^2-q_2q_4S_{I2,10}^2-q_2p_3S_{I2,11}^2+q_2p_3p_4S_{I2,12}^2+q_2p_3q_4S_{I2,13}^2-q_2q_3S_{I2,14}^2+q_2p_4q_3S_{I2,15}^2+q_2q_3q_4S_{I2,16}^2\\
&-p_1p_4S_{I3,9}^2-p_1q_4S_{I3,10}^2-p_1p_3S_{I3,11}^2+p_1p_3p_4S_{I3,12}^2+p_1p_3q_4S_{I3,13}^2-p_1q_3S_{I3,14}^2+p_1p_4q_3S_{I3,15}^2+p_1q_3q_4S_{I3,16}^2+p_1p_2p_4S_{I4,9}^2\\
&+p_1p_2q_4S_{I4,10}^2+p_1p_2p_3S_{I4,11}^2-p_1p_2p_3p_4S_{I4,12}^2-p_1p_2p_3q_4S_{I4,13}^2+p_1p_2q_3S_{I4,14}^2-p_1p_2p_4q_3S_{I4,15}^2-p_1p_2q_3q_4S_{I4,16}^2+p_1q_2p_4S_{I5,9}^2\\
&+p_1q_2q_4S_{I5,10}^2+p_1q_2p_3S_{I5,11}^2-p_1q_2p_3p_4S_{I5,12}^2-p_1q_2p_3q_4S_{I5,13}^2+p_1q_2q_3S_{I5,14}^2-p_1q_2p_4q_3S_{I5,15}^2-p_1q_2q_3q_4S_{I5,16}^2-q_1p_4S_{I6,9}^2\\
&-q_1q_4S_{I6,10}^2-q_1p_3S_{I6,11}^2+q_1p_3p_4S_{I6,12}^2+q_1p_3q_4S_{I6,13}^2-q_1q_3S_{I6,14}^2+q_1p_4q_3S_{I6,15}^2+q_1q_3q_4S_{I6,16}^2+q_1p_2p_4S_{I7,9}^2+q_1p_2q_4S_{I7,10}^2\\
&+q_1p_2p_3S_{I7,11}^2-q_1p_2p_3p_4S_{I7,12}^2-q_1p_2p_3q_4S_{I7,13}^2+q_1p_2q_3S_{I7,14}^2-q_1p_2p_4q_3S_{I7,15}^2-q_1p_2q_3q_4S_{I7,16}^2+q_1q_2p_4S_{I8,9}^2+q_1q_2q_4S_{I8,10}^2\\
&+q_1q_2p_3S_{I8,11}^2-q_1q_2p_3p_4S_{I8,12}^2-q_1q_2p_3q_4S_{I8,13}^2+q_1q_2q_3S_{I8,14}^2-q_1q_2p_4q_3S_{I8,15}^2-q_1q_2q_3q_4S_{I8,16}^2\Big)\Big)-\frac{1}{\lambda^2}(N^2-M)\Big(\frac{p_2}{1-B_{I1}}+\frac{q_2}{1-B_{I2}}\\
&+\frac{p_1}{1-B_{I3}}-\frac{p_1p_2}{1-B_{I4}}-\frac{p_1q_2}{1-B_{I5}}+\frac{q_1}{1-B_{I6}}-\frac{q_1p_2}{1-B_{I7}}-\frac{q_1q_2}{1-B_{I8}}\Big)-\frac{p}{\lambda^2}(N^2-M)\Big(\frac{p_4}{1-B_{I9}}+\frac{q_4}{1-B_{I10}}+\frac{p_3}{1-B_{I11}}-\frac{p_3p_4}{1-B_{I12}}-\frac{p_3q_4}{1-B_{I13}}+\frac{q_3}{1-B_{I14}}+\frac{q_3p_4}{1-B_{I15}}\\
&-\frac{q_3q_4}{1-B_{I16}}-\frac{p_4p_2}{1-B_{I1}B_{I9}}-\frac{p_2q_4}{1-B_{I1}B_{I10}}-\frac{p_2p_3}{1-B_{I1}B_{I11}}+\frac{p_2p_3p_4}{1-B_{I1}B_{I12}}+\frac{p_2p_3q_4}{1-B_{I1}B_{I13}}-\frac{p_2q_3}{1-B_{I1}B_{I14}}+\frac{p_2q_3p_4}{1-B_{I1}B_{I15}}+\frac{p_2q_3q_4}{1-B_{I1}B_{I16}}-\frac{q_2p_4}{1-B_{I2}B_{I9}}-\frac{q_2q_4}{1-B_{I2}B_{I10}}\\
&-\frac{q_2p_3}{1-B_{I2}B_{I11}}+\frac{q_2p_3p_4}{1-B_{I2}B_{I12}}+\frac{q_2p_3q_4}{1-B_{I2}B_{I13}}-\frac{q_2q_3}{1-B_{I2}B_{I14}}+\frac{q_2q_3p_4}{1-B_{I2}B_{I15}}+\frac{q_2q_3q_4}{1-B_{I2}B_{I16}}-\frac{p_4p_1}{1-B_{I3}B_{I9}}-\frac{p_1q_4}{1-B_{I3}B_{I10}}-\frac{p_1p_3}{1-B_{I3}B_{I11}}+\frac{p_1p_3p_4}{1-B_{I3}B_{I12}}+\frac{p_1p_3q_4}{1-B_{I3}B_{I13}}\\
&-\frac{p_1q_3}{1-B_{I3}B_{I14}}+\frac{p_1q_3p_4}{1-B_{I3}B_{I15}}+\frac{p_1q_3q_4}{1-B_{I3}B_{I16}}+\frac{p_1p_4p_2}{1-B_{I4}B_{I9}}+\frac{p_1p_2q_4}{1-B_{I4}B_{I10}}+\frac{p_1p_2p_3}{1-B_{I4}B_{I11}}-\frac{p_1p_2p_3p_4}{1-B_{I4}B_{I12}}-\frac{p_1p_2p_3q_4}{1-B_{I4}B_{I13}}+\frac{p_1p_2q_3}{1-B_{I4}B_{I14}}-\frac{p_1p_2q_3p_4}{1-B_{I4}B_{I15}}-\frac{p_1p_2q_3q_4}{1-B_{I4}B_{I16}}\\
&+\frac{p_1p_4q_2}{1-B_{I5}B_{I9}}+\frac{p_1q_2q_4}{1-B_{I5}B_{I10}}+\frac{p_1q_2p_3}{1-B_{I5}B_{I11}}-\frac{p_1q_2p_3p_4}{1-B_{I5}B_{I12}}-\frac{p_1q_2p_3q_4}{1-B_{I5}B_{I13}}+\frac{p_1q_2q_3}{1-B_{I5}B_{I14}}-\frac{p_1q_2q_3p_4}{1-B_{I5}B_{I15}}-\frac{p_1q_2q_3q_4}{1-B_{I5}B_{I16}}-\frac{p_4q_1}{1-B_{I6}B_{I9}}-\frac{q_1q_4}{1-B_{I6}B_{I10}}-\frac{q_1p_3}{1-B_{I6}B_{I11}}\\
&+\frac{q_1p_3p_4}{1-B_{I6}B_{I12}}+\frac{q_1p_3q_4}{1-B_{I6}B_{I13}}-\frac{q_1q_3}{1-B_{I6}B_{I14}}+\frac{q_1q_3p_4}{1-B_{I6}B_{I15}}+\frac{q_1q_3q_4}{1-B_{I6}B_{I16}}+\frac{q_1p_4p_2}{1-B_{I7}B_{I9}}+\frac{q_1p_2q_4}{1-B_{I7}B_{I10}}+\frac{q_1p_2p_3}{1-B_{I7}B_{I11}}-\frac{q_1p_2p_3p_4}{1-B_{I7}B_{I12}}-\frac{q_1p_2p_3q_4}{1-B_{I7}B_{I13}}+\frac{q_1p_2q_3}{1-B_{I7}B_{I14}}\\
&-\frac{q_1p_2q_3p_4}{1-B_{I7}B_{I15}}-\frac{q_1p_2q_3q_4}{1-B_{I7}B_{I16}}+\frac{q_1p_4q_2}{1-B_{I8}B_{I9}}+\frac{q_1q_2q_4}{1-B_{I8}B_{I10}}+\frac{q_1q_2p_3}{1-B_{I8}B_{I11}}-\frac{q_1q_2p_3p_4}{1-B_{I8}B_{I12}}-\frac{q_1q_2p_3q_4}{1-B_{I8}B_{I13}}+\frac{q_1q_2q_3}{1-B_{I8}B_{I14}}-\frac{q_1q_2q_3p_4}{1-B_{I8}B_{I15}}-\frac{q_1q_2q_3q_4}{1-B_{I8}B_{I16}}\Big)
\end{aligned} \tag{50}$$

where for a=1,2….16, b=1,2,3,4,5,6,7,8 and d=9,10,11,12,13,14,15,16.

$$R_{Ia}=\frac{N}{\lambda(1-B_{Ia})} \text{ and } S_{Ib,d}=\frac{N}{\lambda(1-B_{Ib}B_{Id})} \tag{51}$$

III. MODEL DESCRIPTION AND ANALYSIS OF MODEL-II

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as Y=min (Y_1, Y_2) and Z=min (Z_1, Z_2). All the other assumptions and notations are as in model-I.

Case (i): The distribution of optional and mandatory thresholds follow exponential distribution

For this case the first two moments of time to recruitment are found to be

Proceeding as in model-I, it can be shown for the present model that

If $g(t)=g_{x(1)}(t), f(t)=f_{u(1)}(t)$

$$E(T)=C_{I3}+p(C_{I6}-H_{I3,6}) \tag{52}$$

$$E(T^2)=2\left(C_{I3}^2+p(C_{I6}^2-H_{I3,6}^2)\right) \tag{53}$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6 $C_{Ia}, H_{Ib,d}$ are given by (16)

If $g(t)=g_{x(k)}(t), f(t)=f_{u(1)}(t)$

$$E(T)=L_{K3}+p(L_{K6}-M_{K3,6}) \tag{54}$$

$$E(T^2)=2(L_{K3}^2+p(L_{K6}^2-M_{K3,6}^2)) \tag{55}$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6 $L_{Ka}, M_{Kb,d}$ are given by (20).

If $g(t)=g_{x(k)}(t), f(t)=f_{u(k)}(t)$

$$E(T)=P_{K3}+p(P_{K6}-Q_{K3,6}) \tag{56}$$

$$E(T^2)=2\left(P_{K3}^2+p\left(P_{K6}^2-Q_{K3,6}^2\right)\right)-\frac{1}{\lambda^2}\left(N^2-M\right)\left(\frac{1}{(1-D_{K3})}\right)-\frac{p}{\lambda^2}\left(N^2-M\right)\left(\frac{1}{1-D_{K6}}-\frac{1}{1-D_{K3}D_{K6}}\right) \tag{57}$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6 $P_{Ka}, Q_{Kb,d}$ are given by (24)

If $g(t)=g_{x(1)}(t), f(t)=f_{u(k)}(t)$

$$E(T) = R_{I3} + p(R_{I6} - S_{I3,6}) \tag{58}$$

$$E(T^2) = 2\left(R_{I3}^2 + p\left(R_{I6}^2 - S_{I3,6}^2\right)\right) - \frac{1}{\lambda^2}\left(N^2 - M\right)\left(\frac{1}{(1-D_{I3})}\right) - \frac{p}{\lambda^2}\left(N^2 - M\right)\left(\frac{1}{1-D_{I6}} - \frac{1}{1-D_{I3}D_{I6}}\right) \tag{59}$$

where for a = 1, 2…6. b=1, 2, 3 and d=4, 5, 6 $R_{Ia}, S_{Ib,d}$ are given by (26).

Case (ii): The distribution of optional and mandatory thresholds follow extended exponential distribution

If $g(t) = g_{x(1)}(t), f(t) = f_{u(1)}(t)$

$$E(T) = 4C_{I3} - 2C_{I7} - 2C_{I8} + C_{I11} + p(4C_{I6} - 2C_{I12} - 2C_{I13} + C_{I16} - 16H_{I3,6} + 8H_{I3,12} + 8H_{I3,13} - 4H_{I3,16} + 8H_{I7,6} - 4H_{I7,12} - 4H_{I7,13} + 2H_{I7,6} + 8H_{I8,6} - 4H_{I8,12} - 4H_{I8,13} + 2H_{I8,16} - 4H_{I11,6} - 2H_{I11,12} + 2H_{I11,13} - H_{I11,16}) \tag{60}$$

$$E(T^2) = 2\left(4C_{I3}^2 - 2C_{I7}^2 - 2C_{I8}^2 + C_{I11}^2 + p\left(4C_{I6}^2 - 2C_{I12}^2 - 2C_{I13}^2 + C_{I16}^2 - 16H_{I3,6}^2 + 8H_{I3,12}^2 + 8H_{I3,13}^2 - 4H_{I3,16}^2 + 8H_{I7,6}^2 - 4H_{I7,12}^2 - 4H_{I7,13}^2 + 2H_{I7,16}^2 + 8H_{I8,6}^2 - 4H_{I8,12}^2 - 4H_{I8,13}^2 + 2H_{I8,16}^2 - 4H_{I11,6}^2 - 2H_{I11,12}^2 + 2H_{I11,13}^2 - H_{I11,16}^2\right)\right) \tag{61}$$

where for a=3, 6,7,8,11,12, 13,16 ,b=3,7,8,11,and d=6,12,13,16 $C_{Ia}, H_{Ib,d}$ are given by (16)

If $g(t) = g_{x(k)}(t), f(t) = f_{u(1)}(t)$

$$E(T) = 4L_{K3} - 2L_{K7} - 2L_{K8} + L_{K11} + p(4L_{K6} - 2L_{K12} - 2L_{K13} + L_{K16} - 16M_{K3,6} + 8M_{K3,12} + 8M_{K3,13} - 4M_{K3,16} + 8M_{K7,6} - 4M_{K7,12} - 4M_{K7,13} + 2M_{K7,6} + 8M_{K8,6} - 4M_{K8,12} - 4M_{K8,13} + 2M_{K8,16} - 4M_{K11,6} - 2M_{K11,12} + 2M_{K11,13} - M_{K11,16}) \tag{62}$$

$$E(T^2) = 2\left(4L_{K3}^2 - 2L_{K7}^2 - 2L_{K8}^2 + L_{K11}^2 + p\left(4L_{K6}^2 - 2L_{K12}^2 - 2L_{K13}^2 + L_{K16}^2 - 16L_{K3,6}^2 + 8L_{K3,12}^2 + 8L_{K3,13}^2 - 4L_{K3,16}^2 + 8L_{K7,6}^2 - 4L_{K7,12}^2 - 4L_{K7,13}^2 + 2L_{K7,16}^2 + 8L_{K8,6}^2 - 4L_{K8,12}^2 - 4L_{K8,13}^2 + 2L_{K8,16}^2 - 4L_{K11,6}^2 - 2L_{K11,12}^2 + 2L_{K11,13}^2 - L_{K11,16}^2\right)\right) \tag{63}$$

where for a=3, 6,7,8,11,12, 13,16 ,b=3,7,8,11,and d=6,12,13,16 $L_{Ka}, M_{Kb,d}$ are given by (20).

If $g(t) = g_{x(k)}(t), f(t) = f_{u(k)}(t)$

$$E(T) = 4P_{K3} - 2P_{K7} - 2P_{K8} + P_{K11} + p(4P_{K6} - 2P_{K12} - 2P_{K13} + P_{K16} - 16Q_{K3,6} + 8Q_{K3,12} + 8Q_{K3,13} - 4Q_{K3,16} + 8Q_{K7,6} - 4Q_{K7,12} - 4Q_{K7,13} + 2Q_{K7,6} + 8Q_{K8,6} - 4Q_{K8,12} - 4Q_{K8,13} + 2Q_{K8,16} - 4Q_{K11,6} - 2Q_{K11,12} + 2Q_{K11,13} - Q_{K11,16}) \tag{64}$$

$$\begin{aligned}E(T^2) = {} & 2\left(4P_{K3}^2 - 2P_{K7}^2 - 2P_{K8}^2 + P_{K11}^2 + p\left(4P_{K6}^2 - 2P_{K12}^2 - 2P_{K13}^2 + P_{K16}^2 - 16Q_{K3,6}^2 + 8Q_{K3,12}^2 + 8Q_{K3,13}^2 - 4Q_{K3,16}^2 + 8Q_{K7,6}^2 - 4Q_{K7,12}^2 - 4Q_{K7,13}^2 + 2Q_{K7,16}^2\right.\right. \\ & \left.\left. + 8Q_{K8,6}^2 - 4Q_{K8,12}^2 - 4Q_{K8,13}^2 + 2Q_{K8,16}^2 - 4Q_{K11,6}^2 - 2Q_{K11,12}^2 + 2Q_{K11,13}^2 - Q_{K11,16}^2\right)\right) - \frac{1}{\lambda^2}(N^2 - M)\left(\frac{4}{1-D_{K3}} - \frac{2}{1-D_{K7}} - \frac{2}{1-D_{K8}} + \frac{1}{1-D_{K11}}\right) \\ & - \frac{p}{\lambda^2}(N^2 - M)\left(\frac{4}{1-D_{K6}} - \frac{2}{1-D_{K12}} - \frac{2}{1-D_{K13}} + \frac{1}{1-D_{K16}} - \frac{16}{1-D_{K3}D_{K6}} + \frac{8}{1-D_{K3}D_{K12}} + \frac{8}{1-D_{K3}D_{K13}} - \frac{4}{1-D_{K3}D_{K16}} + \frac{8}{1-D_{K7}D_{K6}} - \frac{4}{1-D_{K7}D_{K12}}\right. \\ & \left. - \frac{4}{1-D_{K7}D_{K13}} + \frac{2}{1-D_{K7}D_{K6}} + \frac{8}{1-D_{K8}D_{K6}} - \frac{4}{1-D_{K8}D_{K12}} - \frac{4}{1-D_{K8}D_{K13}} + \frac{2}{1-D_{K8}D_{K16}} - \frac{4}{1-D_{K11}D_{K6}} - \frac{2}{1-D_{K11}D_{K12}} + \frac{2}{1-D_{K11}D_{K13}} - \frac{1}{1-D_{K11}D_{K16}}\right)\end{aligned} \tag{65}$$

where for a=3, 6,7,8,11,12, 13,16 ,b=3,7,8,11,and d=6,12,13,16 $P_{Ka}, Q_{Kb,d}$ are given by (24) .

If $g(t) = g_{x(1)}(t), f(t) = f_{u(k)}(t)$

$$E(T) = 4R_{I3} - 2R_{I7} - 2R_{I8} + R_{I11} + p(4R_{I6} - 2R_{I12} - 2R_{I13} + R_{I16} - 16S_{I3,6} + 8S_{I3,12} + 8S_{I3,13} - 4S_{I3,16} + 8S_{I7,6} - 4S_{I7,12} - 4S_{I7,13} + 2S_{I7,6} + 8S_{I8,6} - 4S_{I8,12} - 4S_{I8,13} + 2S_{I8,16} - 4S_{I11,6} - 2S_{I11,12} + 2S_{I11,13} - S_{I11,16}) \tag{66}$$

$$\begin{aligned}E(T^2) = {} & 2\left(4R_{I3}^2 - 2R_{I7}^2 - 2R_{I8}^2 + R_{I11}^2 + p\left(4R_{I6}^2 - 2R_{I12}^2 - 2R_{I13}^2 + R_{I16}^2 - 16S_{I3,6}^2 + 8S_{I3,12}^2 + 8S_{I3,13}^2 - 4S_{I3,16}^2 + 8S_{I7,6}^2 - 4S_{I7,12}^2 - 4S_{I7,13}^2 + 2S_{I7,16}^2 + 8S_{I8,6}^2\right.\right. \\ & \left.\left. - 4S_{I8,12}^2 - 4S_{I8,13}^2 + 2S_{I8,16}^2 - 4S_{I11,6}^2 - 2S_{I11,12}^2 + 2S_{I11,13}^2 - S_{I11,16}^2\right)\right) - \frac{1}{\lambda^2}(N^2 - M)\left(\frac{4}{1-D_{I3}} - \frac{2}{1-D_{I7}} - \frac{2}{1-D_{I8}} + \frac{1}{1-D_{I11}}\right) - \frac{p}{\lambda^2}(N^2 - M)\left(\frac{4}{1-D_{I6}} - \frac{2}{1-D_{I12}}\right. \\ & - \frac{2}{1-D_{I13}} + \frac{1}{1-D_{I16}} - \frac{16}{1-D_{I3}D_{I6}} + \frac{8}{1-D_{I3}D_{I12}} + \frac{8}{1-D_{I3}D_{I13}} - \frac{4}{1-D_{I3}D_{I16}} + \frac{8}{1-D_{I7}D_{I6}} - \frac{4}{1-D_{I7}D_{I12}} - \frac{4}{1-D_{I7}D_{I13}} + \frac{2}{1-D_{I7}D_{IK6}} + \frac{8}{1-D_{I8}D_{I6}} \\ & \left. - \frac{4}{1-D_{I8}D_{I12}} - \frac{4}{1-D_{I8}D_{I13}} + \frac{2}{1-D_{I8}D_{I16}} - \frac{4}{1-D_{I11}D_{I6}} - \frac{2}{1-D_{I11}D_{I12}} + \frac{2}{1-D_{I11}D_{I13}} - \frac{1}{1-D_{I11}D_{I16}}\right)\end{aligned} \tag{67}$$

where for a=3, 6,7,8,11,12, 13,16 ,b=3,7,8,11,and d=6,12,13,16 $R_{Ia}, S_{Ib,d}$ are given by (26).

Case (iii): The distributions of optional and mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t), f(t) = f_{u(1)}(t)$

$$\begin{aligned}E(T) = {} & p_1p_2C_{I4} + p_1q_2C_{I5} + p_2q_1C_{I7} + q_1q_2C_{I8} + p(p_3p_4C_{I12} + p_3q_4C_{I13} + p_4q_3C_{I15} + q_3q_4C_{I16} \\ & - p_1p_2p_3p_4H_{I4,12} - p_1p_2p_3q_4H_{I4,13} - p_1p_2q_3p_4H_{I4,15} - p_1p_2q_3q_4H_{I4,16} - p_1q_2p_3p_4H_{I5,12} \\ & - p_1q_2p_3q_4H_{I5,13} - p_1q_2p_4q_3H_{I5,15} - p_1q_2q_3q_4H_{I5,16} - q_1p_2p_3p_4H_{I7,12} - q_1p_2p_3q_4H_{I7,13} \\ & - q_1p_2p_4q_3H_{I7,15} - q_1p_2q_3q_4H_{I7,16} - q_1q_2p_3p_4H_{I8,12} - q_1q_2p_3q_4H_{I8,13} - q_1q_2p_4q_3H_{I8,15} - q_1q_2q_3q_4H_{I8,16})\end{aligned} \tag{68}$$

$$E(T^2) = 2\Big(p_1p_2C^2_{I4} + p_1q_2C^2_{I5} + p_2q_1C^2_{I7} + q_1q_2C^2_{I8} + p\Big(p_3p_4C^2_{I12} + p_3q_4C^2_{I13} + p_4q_3C^2_{I15} + q_3q_4C^2_{I16}$$
$$- p_1p_2p_3p_4H^2_{I4,12} - p_1p_2p_3q_4H^2_{I4,13} - p_1p_2p_4q_3H^2_{I4,15} - p_1p_2q_3q_4H^2_{I4,16} - p_1q_2p_3p_4H^2_{I5,12}$$
$$- p_1q_2p_3q_4H^2_{I5,13} - p_1q_2p_4q_3H^2_{I5,15} - p_1q_2q_3q_4H^2_{I5,16} - q_1p_2p_3p_4H^2_{I7,12} - q_1p_2p_3q_4H^2_{I7,13} - q_1p_2p_4q_3H^2_{I7,15}$$
$$- q_1p_2q_3q_4H^2_{I7,16} - q_1q_2p_3p_4H^2_{I8,12} - q_1q_2p_3q_4H^2_{I8,13} + q_1q_2q_3H^2_{I8,14} - q_1q_2p_4q_3H^2_{I8,15} - q_1q_2q_3q_4H^2_{I8,16}\Big)\Big) \quad (69)$$

where for a=4,5,7,8, 12, 13,15,16 ,b=4,5,7,8 and d=12, 13,15,16. $C_{Ia}, H_{Ib,d}$ are given by (39) .

If $g(t) = g_{x(k)}(t), f(t) = f_{u(1)}(t)$

$$E(T) = p_1p_2L_{K4} + p_1q_2L_{K5} + p_2q_1L_{K7} + q_1q_2L_{K8} + p(p_3p_4L_{K12} + p_3q_4L_{K13} + p_4q_3L_{K15} + q_3q_4L_{K16} \quad (70)$$
$$- p_1p_2p_3p_4M_{K4,12} - p_1p_2p_3q_4M_{K4,13} - p_1p_2q_3p_4M_{K4,15} - p_1p_2q_3q_4M_{K4,16} - p_1q_2p_3p_4M_{K5,12}$$
$$- p_1q_2p_3q_4M_{K5,13} - p_1q_2p_4q_3M_{K5,15} - p_1q_2q_3q_4M_{K5,16} - q_1p_2p_3p_4M_{K7,12} - q_1p_2p_3q_4M_{K7,13}$$
$$- q_1p_2p_4q_3M_{K7,15} - q_1p_2q_3q_4M_{K7,16} - q_1q_2p_3p_4M_{K8,12} - q_1q_2p_3q_4M_{K8,13} - q_1q_2p_4q_3M_{K8,15} - q_1q_2q_3q_4M_{K8,16})$$

$$E(T^2) = 2\Big(p_1p_2L^2_{K4} + p_1q_2L^2_{K5} + p_2q_1L^2_{K7} + q_1q_2L^2_{K8} + p\Big(p_3p_4L^2_{K12} + p_3q_4L^2_{K13} + p_4q_3L^2_{K15} + q_3q_4L^2_{K16} - p_1p_2p_3p_4M^2_{K4,12}$$
$$- p_1p_2p_3q_4M^2_{K4,13} - p_1p_2p_4q_3M^2_{K4,15} - p_1p_2q_3q_4M^2_{K4,16} - p_1q_2p_3p_4M^2_{K5,12} - p_1q_2p_3q_4M^2_{K5,13} - p_1q_2p_4q_3M^2_{K5,15}$$
$$- p_1q_2q_3q_4M^2_{K5,16} - q_1p_2p_3p_4M^2_{K7,12} - q_1p_2p_3q_4M^2_{K7,13} - q_1p_2p_4q_3M^2_{K7,15} - q_1p_2q_3q_4M^2_{K7,16} - q_1q_2p_3p_4M^2_{K8,12}$$
$$- q_1q_2p_3q_4M^2_{K8,13} + q_1q_2q_3M^2_{K8,14} - q_1q_2p_4q_3M^2_{K8,15} - q_1q_2q_3q_4M^2_{K8,16}\Big)\Big) \quad (71)$$

where for a=4,5,7,8, 12, 13,15,16 ,b=4,5,7,8 and d=12, 13,15,16. $L_{Ka}, M_{Kb,d}$ are given by (44).

If $g(t) = g_{x(k)}(t), f(t) = f_{u(k)}(t)$

$$E(T) = p_1p_2P_{K4} + p_1q_2P_{K5} + p_2q_1P_{K7} + q_1q_2P_{K8} + p(p_3p_4P_{K12} + p_3q_4P_{K13} + p_4q_3P_{K15} + q_3q_4P_{K16}$$
$$- p_1p_2p_3p_4Q_{K4,12} - p_1p_2p_3q_4Q_{K4,13} - p_1p_2q_3p_4Q_{K4,15} - p_1p_2q_3q_4Q_{K4,16} - p_1q_2p_3p_4Q_{K5,12}$$
$$- p_1q_2p_3q_4Q_{K5,13} - p_1q_2p_4q_3Q_{K5,15} - p_1q_2q_3q_4Q_{K5,16} - q_1p_2p_3p_4Q_{K7,12} - q_1p_2p_3q_4Q_{K7,13}$$
$$- q_1p_2p_4q_3Q_{K7,15} - q_1p_2q_3q_4Q_{K7,16} - q_1q_2p_3p_4Q_{K8,12} - q_1q_2p_3q_4Q_{K8,13} - q_1q_2p_4q_3Q_{K8,15} - q_1q_2q_3q_4Q_{K8,16}) \quad (72)$$

$$E(T^2) = 2\Big(p_1p_2P^2_{K4} + p_1q_2P^2_{K5} + p_2q_1P^2_{K7} + q_1q_2P^2_{K8} + p\Big(p_3p_4P^2_{K12} + p_3q_4P^2_{K13} + p_4q_3P^2_{K15} + q_3q_4P^2_{K16} - p_1p_2p_3p_4Q^2_{K4,12}$$
$$- p_1p_2p_3q_4Q^2_{K4,13} - p_1p_2p_4q_3Q^2_{K4,15} - p_1p_2q_3q_4Q^2_{K4,16} - p_1q_2p_3p_4Q^2_{K5,12} - p_1q_2p_3q_4Q^2_{K5,13} - p_1q_2p_4q_3Q^2_{K5,15}$$
$$- p_1q_2q_3q_4Q^2_{K5,16} - q_1p_2p_3p_4Q^2_{K7,12} - q_1p_2p_3q_4Q^2_{K7,13} - q_1p_2p_4q_3Q^2_{K7,15} - q_1p_2q_3q_4Q^2_{K7,16} - q_1q_2p_3p_4Q^2_{K8,12}$$
$$- q_1q_2p_3q_4Q^2_{K8,13} + q_1q_2q_3Q^2_{K8,14} - q_1q_2p_4q_3Q^2_{K8,15} - q_1q_2q_3q_4Q^2_{K8,16}\Big)\Big) - \frac{1}{\lambda^2}(N^2 - M)\left(\frac{p_1p_2}{1-D_{K4}} + \frac{p_1q_2}{1-D_{K5}} + \frac{q_1p_2}{1-D_{K7}}\right.$$
$$\left. + \frac{q_1q_2}{1-D_{K8}}\right) - \frac{p}{\lambda^2}(N^2 - M)\left(\frac{p_3p_4}{1-D_{K12}} + \frac{p_3q_4}{1-D_{K13}} + \frac{q_3p_4}{1-D_{K15}} + \frac{q_3q_4}{1-D_{K16}} - \frac{p_1p_2p_3p_4}{1-D_{K4}D_{K12}} - \frac{p_1p_2p_3q_4}{1-D_{K4}D_{K13}} - \frac{p_1p_2q_3p_4}{1-D_{K4}D_{K15}}\right.$$
$$- \frac{p_1p_2q_3q_4}{1-D_{K4}D_{K16}} - \frac{p_1q_2p_3p_4}{1-D_{K5}D_{K12}} - \frac{p_1q_2p_3q_4}{1-D_{K5}D_{K13}} - \frac{p_1q_2q_3p_4}{1-D_{K5}D_{K15}} - \frac{p_1q_2q_3q_4}{1-D_{K5}D_{K16}} - \frac{q_1p_2p_3p_4}{1-D_{K7}D_{K12}} - \frac{q_1p_2p_3q_4}{1-D_{K7}D_{K13}}$$
$$\left. - \frac{q_1p_2q_3p_4}{1-D_{K7}D_{K15}} - \frac{q_1p_2q_3q_4}{1-D_{K7}D_{K16}} - \frac{q_1q_2p_3p_4}{1-D_{K8}D_{K12}} - \frac{q_1q_2p_3q_4}{1-D_{K8}D_{K13}} - \frac{q_1q_2q_3p_4}{1-D_{K8}D_{K15}} - \frac{q_1q_2q_3q_4}{1-D_{K8}D_{K16}}\right) \quad (73)$$

where for a=4,5,7,8, 12, 13,15,16 ,b=4,5,7,8 and d=12, 13,15,16. $P_{Ka}, Q_{Kb,d}$ are given by (48).

If $g(t) = g_{x(1)}(t), f(t) = f_{u(k)}(t)$

$$E(T) = p_1p_2R_{I4} + p_1q_2R_{I5} + p_2q_1R_{I7} + q_1q_2R_{I8} + p(p_3p_4R_{I12} + p_3q_4R_{I13} + p_4q_3R_{I15} + q_3q_4R_{I16}$$
$$- p_1p_2p_3p_4S_{I4,12} - p_1p_2p_3q_4S_{I4,13} - p_1p_2q_3p_4S_{I4,15} - p_1p_2q_3q_4S_{I4,16} - p_1q_2p_3p_4S_{I5,12}$$
$$- p_1q_2p_3q_4S_{I5,13} - p_1q_2p_4q_3S_{I5,15} - p_1q_2q_3q_4S_{I5,16} - q_1p_2p_3p_4S_{I7,12} - q_1p_2p_3q_4S_{I7,13}$$
$$- q_1p_2p_4q_3S_{I7,15} - q_1p_2q_3q_4S_{I7,16} - q_1q_2p_3p_4S_{I8,12} - q_1q_2p_3q_4S_{I8,13} - q_1q_2p_4q_3S_{I8,15} - q_1q_2q_3q_4S_{I8,16}) \quad (74)$$

$$E(T^2)=2\Big(p_1p_2R_{I4}^2+p_1q_2R_{I5}^2+p_2q_1R_{I7}^2+q_1q_2R_{I8}^2+p\Big(p_3p_4R_{I12}^2+p_3q_4R_{I13}^2+p_4q_3R_{I15}^2+q_3q_4R_{I16}^2-p_1p_2p_3p_4S_{I4,12}^2-p_1p_2p_3q_4S_{I4,13}^2$$
$$-p_1p_2p_4q_3S_{I4,15}^2-p_1p_2q_3q_4S_{I4,16}^2-p_1q_2p_3p_4S_{I5,12}^2-p_1q_2p_3q_4S_{I5,13}^2-p_1q_2p_4q_3S_{I5,15}^2-p_1q_2q_3q_4S_{I5,16}^2-q_1p_2p_3p_4S_{I7,12}^2$$
$$-q_1p_2p_3q_4S_{I7,13}^2-q_1p_2p_4q_3S_{I7,15}^2-q_1p_2q_3q_4S_{I7,16}^2-q_1q_2p_3p_4S_{I8,12}^2-q_1q_2p_3q_4S_{I8,13}^2+q_1q_2q_3S_{I8,14}^2-q_1q_2p_4q_3S_{I8,15}^2$$
$$-q_1q_2q_3q_4S_{I8,16}^2\Big)\Big)-\frac{1}{\lambda^2}(N^2-M)\left(\frac{p_1p_2}{1-D_{I4}}+\frac{p_1q_2}{1-D_{I5}}+\frac{q_1p_2}{1-D_{I7}}+\frac{q_1q_2}{1-D_{I8}}\right)-\frac{p}{\lambda^2}(N^2-M)\left(\frac{p_3p_4}{1-D_{I12}}+\frac{p_3q_4}{1-D_{I13}}+\frac{q_3p_4}{1-D_{I15}}+\frac{q_3q_4}{1-D_{I16}}\right.$$
$$-\frac{p_1p_2p_3p_4}{1-D_{I4}D_{I12}}-\frac{p_1p_2p_3q_4}{1-D_{I4}D_{I13}}-\frac{p_1p_2q_3p_4}{1-D_{I4}D_{I5}}-\frac{p_1p_2q_3q_4}{1-D_{I4}D_{I16}}-\frac{p_1q_2p_3p_4}{1-D_{I5}D_{I12}}-\frac{p_1q_2p_3q_4}{1-D_{I5}D_{I13}}-\frac{p_1q_2q_3p_4}{1-D_{I5}D_{I15}}-\frac{p_1q_2q_3q_4}{1-D_{I5}D_{I16}}$$
$$\left.-\frac{q_1p_2p_3p_4}{1-D_{I7}D_{I12}}-\frac{q_1p_2p_3q_4}{1-D_{I7}D_{I13}}-\frac{q_1p_2q_3p_4}{1-D_{I7}D_{I15}}-\frac{q_1p_2q_3q_4}{1-D_{I7}D_{I16}}-\frac{q_1q_2p_3p_4}{1-D_{I8}D_{I12}}-\frac{q_1q_2p_3q_4}{1-D_{I8}D_{I13}}-\frac{q_1q_2q_3p_4}{1-D_{I8}D_{I15}}-\frac{q_1q_2q_3q_4}{1-D_{I8}D_{I16}}\right) \quad (75)$$

where for a=4,5,7,8, 12, 13,15,16 ,b=4,5,7,8 and d=12, 13,15,16. $R_{Ia}, S_{Ib,d}$ are given by (51).

IV. MODEL DESCRIPTION AND ANALYSIS OF MODEL-III

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y=Y_1+Y_2$ and $Z=Z_1+Z_2$ .All the other assumptions and notations are as in model-I. Proceeding as in model-I, it can be shown for the present model that

Case (i): The distributions of optional and mandatory thresholds follow exponential distribution.

If $g(t)=g_{x(1)}(t), f(t)=f_{u(1)}(t)$

$$E(T)=A_2C_{I2}-A_1C_{I1}+p\left(A_5C_{I5}-A_4C_{I4}-A_1A_4H_{I1,4}+A_2A_4H_{I1,4}+A_1A_5H_{I1,5}-A_2A_5H_{I2,5}\right) \quad (76)$$

$$E(T^2)=2\left(A_2C_{I2}^2-A_1C_{I1}^2+p\left(A_5C_{I5}^2-A_4C_{I4}^2-A_1A_4H_{I1,4}^2+A_2A_4H_{I1,4}^2+A_1A_5H_{I1,5}^2-A_2A_5H_{I2,5}^2\right)\right) \quad (77)$$

where

$$A_1=\frac{\theta_2}{\theta_1-\theta_2}, A_2=\frac{\theta_1}{\theta_1-\theta_2}, A_4=\frac{\alpha_2}{\alpha_1-\alpha_2}, A_5=\frac{\alpha_1}{\alpha_1-\alpha_2}$$

for a=1, 2, 4, 5, b=1,2and d=4, 5 $C_{Ia}, H_{Ib,d}$ are given by equation (16).

If $g(t)=g_{x(k)}(t), f(t)=f_{u(1)}(t)$

$$E(T)=A_2L_{K2}-A_1L_{K1}+p\left(A_5L_{K5}-A_4L_{K4}-A_1A_4M_{K1,4}+A_2A_4M_{K1,4}+A_1A_5M_{K1,5}-A_2A_5M_{K2,5}\right) \quad (78)$$

$$E(T^2)=2\left(A_2L_{K2}^2-A_1L_{K1}^2+p\left(A_5L_{K5}^2-A_4L_{K4}^2-A_1A_4M_{K1,4}^2+A_2A_4M_{K1,4}^2+A_1A_5M_{K1,5}^2-A_2A_5M_{K2,5}^2\right)\right) \quad (79)$$

where for a=1, 2, 4, 5, b=1,2and d=4, 5 $L_{Ka}, M_{Kb,d}$ are given by (20).

If $g(t)=g_{x(k)}(t), f(t)=f_{u(k)}(t)$

$$E(T)=A_2P_{K2}-A_1P_{K1}+p\left(A_5P_{K5}-A_4P_{K4}-A_1A_4Q_{K1,4}+A_2A_4Q_{K1,4}+A_1A_5Q_{K1,5}-A_2A_5Q_{K2,5}\right) \quad (80)$$

$$E(T^2)=2\left(A_2P_{K2}^2-A_1P_{K1}^2+p\left(A_5P_{K5}^2-A_4P_{K4}^2-A_1A_4Q_{K1,4}^2+A_2A_4Q_{K1,4}^2+A_1A_5Q_{K1,5}^2-A_2A_5Q_{K2,5}^2\right)\right)-\frac{1}{\lambda^2}(N^2-M)\left(\frac{A_2}{1-D_{K2}}-\frac{A_1}{1-D_{K1}}\right)$$
$$-\frac{p}{\lambda^2}(N^2-M)\left(\frac{A_5}{1-D_{K5}}-\frac{A_4}{1-D_{K4}}-\frac{A_1A_4}{1-D_{K1}D_{K4}}+\frac{A_2A_4}{1-D_{K1}D_{K4}}+\frac{A_1A_5}{1-D_{K1}D_{K5}}-\frac{A_2A_5}{1-D_{K2}D_{K5}}\right) \quad (81)$$

where for a=1, 2, 4, 5, b=1,2and d=4, 5 $P_{Ka}, Q_{Kb,d}$ are given by (24) .

If $g(t)=g_{x(1)}(t), f(t)=f_{u(k)}(t)$

$$E(T)=A_2R_{I2}-A_1R_{I1}+p\left(A_5R_{I5}-A_4R_{I4}-A_1A_4S_{I1,4}+A_2A_4S_{I1,4}+A_1A_5S_{I1,5}-A_2A_5S_{I2,5}\right) \quad (82)$$

$$E(T^2)=2\left(A_2R_{I2}^2-A_1R_{I1}^2+p\left(A_5R_{I5}^2-A_4R_{I4}^2-A_1A_4S_{I1,4}^2+A_2A_4S_{I1,4}^2+A_1A_5S_{I1,5}^2-A_2A_5S_{I2,5}^2\right)\right)$$
$$-\frac{1}{\lambda^2}(N^2-M)\left(\frac{A_2}{1-D_{I2}}-\frac{A_1}{1-D_{I1}}\right)-\frac{p}{\lambda^2}(N^2-M)\left(\frac{A_5}{1-D_{I5}}-\frac{A_4}{1-D_{I4}}-\frac{A_1A_4}{1-D_{I1}D_{I4}}+\frac{A_2A_4}{1-D_{I1}D_{I4}}+\frac{A_1A_5}{1-D_{I1}D_{I5}}-\frac{A_2A_5}{1-D_{I2}D_{I5}}\right) \quad (83)$$

where for a=1, 2, 4, 5, b=1,2and d=4, 5 $R_{Ia}, S_{Ib,d}$ are given by (27).

Case (ii): If the distributions of optional and mandatory thresholds follow extended exponential distribution with shape parameter 2.

If $g(t)=g_{x(1)}(t), f(t)=f_{u(1)}(t)$

$$E(T)=S_1C_{I1}-S_2C_{I9}+S_3C_{I2}-S_4C_{I10}+p\left(S_5C_{I4}-S_6C_{I14}+S_7C_{I5}-S_8C_{I5}-S_1S_5H_{I1,4}+S_1S_6H_{I1,14}-S_1S_7H_{I1,5}+S_1S_8H_{I1,15}+S_2S_5H_{I9,4}-S_2S_6H_{I9,14}\right.$$
$$\left.+S_2S_7H_{I9,5}-S_2S_8H_{I9,15}-S_3S_5H_{I2,4}+S_3S_6H_{I2,14}-S_3S_7H_{I2,5}+S_3S_8H_{I2,15}+S_4S_5H_{I4,10}-S_4S_6H_{I10,14}+S_4S_7H_{I10,5}-S_4S_8H_{I10,15}\right) \quad (84)$$

$$E(T^2)=2\Big(S_1C_{I1}^{\;2}-S_2C_{I9}^{\;2}+S_3C_{I2}^{\;2}-S_4C_{I10}^{\;2}+p\Big(S_5C_{I4}^{\;2}-S_6C_{I14}^{\;2}+S_7C_{I5}^{\;2}-S_8C_{I5}^{\;2}-S_1S_5H_{I1,4}^{\;2}+S_1S_6H_{I1,14}^{\;2}-S_1S_7H_{I1,5}^{\;2}+S_1S_8H_{I1,15}^{\;2}+S_2S_5H_{I9,4}^{\;2}$$
$$-S_2S_6H_{I9,14}^{\;2}+S_2S_7H_{I9,5}^{\;2}-S_2S_8H_{I9,15}^{\;2}-S_3S_5H_{I2,4}^{\;2}+S_3S_6H_{I2,14}^{\;2}-S_3S_7H_{I2,5}^{\;2}+S_3S_8H_{I2,15}^{\;2}+S_4S_5H_{I4,10}^{\;2}-S_4S_6H_{I10,14}^{\;2}+S_4S_7H_{I10,5}^{\;2}-S_4S_8H_{I10,15}^{\;2}\Big)\Big) \quad (85)$$

Where for a=1, 2, 4, 5, 9,10,14,15 b=1, 2,9,10 and d=4, 5, 14, 15. $C_{Ia}, H_{Ib,d}$ are given by (15)

$$S_1=\frac{4\theta_2^2}{(\theta_1-\theta_2)(\theta_1-2\theta_2)},\ S_2=\frac{\theta_2^2}{(\theta_1-\theta_2)(2\theta_1-\theta_2)},\ S_3=\frac{4\theta_1^2}{(\theta_1-\theta_2)(2\theta_1-\theta_2)}\ S_4=\frac{\theta_1^2}{(\theta_1-\theta_2)(\theta_1-2\theta_2)} \quad (86)$$
$$S_5=\frac{4\alpha_2^2}{(\alpha_1-\alpha_2)(\alpha_1-2\alpha_2)},\ S_6=\frac{\alpha_2^2}{(\alpha_1-\alpha_2)(2\alpha_1-\alpha_2)}\ S_7=\frac{4\alpha_1^2}{(\alpha_1-\alpha_2)(2\alpha_1-\alpha_2)},\ S_8=\frac{\alpha_1^2}{(\alpha_1-\alpha_2)(\alpha_1-2\alpha_2)}$$

If $g(t)=g_{x(k)}(t), f(t)=f_{u(1)}(t)$

$$E(T)=S_1L_{K1}-S_2L_{K9}+S_3L_{K2}-S_4L_{K10}+p\big(S_5L_{K4}-S_6L_{K14}+S_7L_{K5}-S_8L_{K5}-S_1S_5M_{K1,4}+S_1S_6M_{K1,14}$$
$$-S_1S_7M_{K1,5}+S_1S_8M_{K1,15}+S_2S_5M_{K9,4}-S_2S_6M_{K9,14}+S_2S_7M_{K9,5}-S_2S_8M_{K9,15}-S_3S_5M_{K2,4}+S_3S_6M_{K2,14}$$
$$-S_3S_7M_{K2,5}+S_3S_8M_{K2,15}+S_4S_5M_{K4,10}-S_4S_6M_{K10,14}+S_4S_7M_{K10,5}-S_4S_8M_{K10,15}\big) \quad (87)$$

$$E(T^2)=2\Big(S_1L_{K1}^{\;2}-S_2L_{K9}^{\;2}+S_3L_{K2}^{\;2}-S_4L_{K10}^{\;2}+p\Big(S_5L_{K4}^{\;2}-S_6L_{K14}^{\;2}+S_7L_{K5}^{\;2}-S_8L_{K15}^{\;2}-S_1S_5M_{K1,4}^{\;2}+S_1S_6M_{K1,14}^{\;2}$$
$$-S_1S_7M_{K1,5}^{\;2}+S_1S_8M_{K1,15}^{\;2}+S_2S_5M_{K9,4}^{\;2}-S_2S_6M_{K9,14}^{\;2}+S_2S_7M_{K9,5}^{\;2}-S_2S_8M_{K9,15}^{\;2}-S_3S_5M_{K2,4}^{\;2}+S_3S_6M_{K2,14}^{\;2}$$
$$-S_3S_7M_{K2,5}^{\;2}+S_3S_8M_{K2,15}^{\;2}+S_4S_5M_{K4,10}^{\;2}-S_4S_6M_{K10,14}^{\;2}+S_4S_7M_{K10,5}^{\;2}-S_4S_8M_{K10,15}^{\;2}\Big)\Big) \quad (88)$$

where for a=1, 2,4,5 ,9,10,14,15 b=1,2,9,10 and d=4,5,14,15 $L_{Ka}, M_{Kb,d}$ are given by (20).

If $g(t)=g_{x(k)}(t), f(t)=f_{u(k)}(t)$

$$E(T)=S_1P_{K1}-S_2P_{K9}+S_3P_{K2}-S_4P_{K10}+p\big(S_5P_{K4}-S_6P_{K14}+S_7P_{K5}-S_8P_{K5}-S_1S_5Q_{K1,4}+S_1S_6Q_{K1,14}$$
$$-S_1S_7Q_{K1,5}+S_1S_8Q_{K1,15}+S_2S_5Q_{K9,4}-S_2S_6Q_{K9,14}+S_2S_7Q_{K9,5}-S_2S_8Q_{K9,15}-S_3S_5Q_{K2,4}+S_3S_6Q_{K2,14}$$
$$-S_3S_7Q_{K2,5}+S_3S_8Q_{K2,15}+S_4S_5Q_{K4,10}-S_4S_6Q_{K10,14}+S_4S_7Q_{K10,5}-S_4S_8Q_{K10,15}\big) \quad (89)$$

$$E(T^2)=2\Big(S_1P_{K1}^{\;2}-S_2P_{K9}^{\;2}+S_3P_{K2}^{\;2}-S_4P_{K10}^{\;2}+p\Big(S_5P_{K4}^{\;2}-S_6P_{K14}^{\;2}+S_7P_{K5}^{\;2}-S_8P_{K15}^{\;2}-S_1S_5Q_{K1,4}^{\;2}+S_1S_6Q_{K1,14}^{\;2}$$
$$-S_1S_7Q_{K1,5}^{\;2}+S_1S_8Q_{K1,15}^{\;2}+S_2S_5Q_{K9,4}^{\;2}-S_2S_6Q_{K9,14}^{\;2}+S_2S_7Q_{K9,5}^{\;2}-S_2S_8Q_{K9,15}^{\;2}-S_3S_5Q_{K2,4}^{\;2}+S_3S_6Q_{K2,14}^{\;2}$$
$$-S_3S_7Q_{K2,5}^{\;2}+S_3S_8Q_{K2,15}^{\;2}+S_4S_5Q_{K4,10}^{\;2}-S_4S_6Q_{K10,14}^{\;2}+S_4S_7Q_{K10,5}^{\;2}-S_4S_8Q_{K10,15}^{\;2}\Big)\Big)-\frac{1}{\lambda^2}(N^2-M)\Big(\frac{S_1}{1-D_{K1}}-$$
$$\frac{S_2}{1-D_{K9}}+\frac{S_3}{1-D_{K2}}-\frac{S_4}{1-D_{K10}}\Big)-\frac{p}{\lambda^2}(N^2-M)\Big(\frac{S_5}{1-D_{K4}}-\frac{S_6}{1-D_{K14}}+\frac{S_7}{1-D_{K5}}-\frac{S_8}{1-D_{K15}}-\frac{S_1S_5}{1-D_{K1}D_{K4}}+\frac{S_1S_6}{1-D_{K1}D_{K14}}$$
$$-\frac{S_1S_7}{1-D_{K1}D_{K5}}+\frac{S_1S_8}{1-D_{K1}D_{K15}}+\frac{S_2S_5}{1-D_{K9}D_{K4}}-\frac{S_2S_6}{1-D_{K9}D_{K14}}+\frac{S_2S_7}{1-D_{K9}D_{K5}}-\frac{S_2S_8}{1-D_{K9}D_{K15}}-\frac{S_3S_5}{1-D_{K2}D_{K4}}+\frac{S_3S_6}{1-D_{K2}D_{K14}}$$
$$-\frac{S_3S_7}{1-D_{K2}D_{K5}}+\frac{S_3S_8}{1-D_{K2}D_{K15}}+\frac{S_4S_5}{1-D_{K10}D_{K4}}-\frac{S_4S_6}{1-D_{K10}D_{K14}}+\frac{S_4S_7}{1-D_{K10}D_{K5}}-\frac{S_4S_8}{1-D_{K10}D_{K15}}\Big) \quad (90)$$

where for a=1, 2,4,5 ,9,10,14,15 b=1,2,9,10 and d=4,5,14,15 $P_{Ka}, Q_{Kb,d}$ are given by (24) .

If $g(t)=g_{x(1)}(t), f(t)=f_{u(k)}(t)$

$$E(T)=S_1R_{I1}-S_2R_{I9}+S_3R_{I2}-S_4R_{I10}+p\big(S_5R_{I4}-S_6R_{I14}+S_7R_{I5}-S_8R_{I5}-S_1S_5S_{I1,4}+S_1S_6S_{I1,14}$$
$$-S_1S_7S_{I1,5}+S_1S_8S_{I1,15}+S_2S_5S_{I9,4}-S_2S_6S_{I9,14}+S_2S_7S_{I9,5}-S_2S_8S_{I9,15}-S_3S_5S_{I2,4}+S_3S_6S_{I2,14}$$
$$-S_3S_7S_{I2,5}+S_3S_8S_{I2,15}+S_4S_5S_{I4,10}-S_4S_6S_{I10,14}+S_4S_7S_{I10,5}-S_4S_8S_{I10,15}\big) \quad (91)$$

$$E(T^2)=2\Big(S_1R_{I1}^{\;2}-S_2R_{I9}^{\;2}+S_3R_{I2}^{\;2}-S_4R_{I10}^{\;2}+p\Big(S_5R_{I4}^{\;2}-S_6R_{I14}^{\;2}+S_7R_{I5}^{\;2}-S_8R_{I5}^{\;2}-S_1S_5S_{I1,4}^{\;2}+S_1S_6S_{I1,14}^{\;2}$$
$$-S_1S_7S_{I1,5}^{\;2}+S_1S_8S_{I1,15}^{\;2}+S_2S_5S_{I9,4}^{\;2}-S_2S_6S_{I9,14}^{\;2}+S_2S_7S_{I9,5}^{\;2}-S_2S_8S_{I9,15}^{\;2}-S_3S_5S_{I2,4}^{\;2}+S_3S_6S_{I2,14}^{\;2}$$
$$-S_3S_7S_{I2,5}^{\;2}+S_3S_8S_{I2,15}^{\;2}+S_4S_5S_{I4,10}^{\;2}-S_4S_6S_{I10,14}^{\;2}+S_4S_7S_{I10,5}^{\;2}-S_4S_8S_{I10,15}^{\;2}\Big)\Big)-\frac{1}{\lambda^2}(N^2-M)\Big(\frac{S_1}{1-D_{I1}}$$
$$-\frac{S_2}{1-D_{I9}}+\frac{S_3}{1-D_{I2}}-\frac{S_4}{1-D_{I10}}\Big)-\frac{p}{\lambda^2}(N^2-M)\Big(\frac{S_5}{1-D_{I4}}-\frac{S_6}{1-D_{I14}}+\frac{S_7}{1-D_{I5}}-\frac{S_8}{1-D_{I15}}-\frac{S_1S_5}{1-D_{I1}D_{I4}}+\frac{S_1S_6}{1-D_{I1}D_{I14}}$$
$$-\frac{S_1S_7}{1-D_{I1}D_{I5}}+\frac{S_1S_8}{1-D_{I1}D_{I15}}+\frac{S_2S_5}{1-D_{I9}D_{I4}}-\frac{S_2S_6}{1-D_{I9}D_{I14}}+\frac{S_2S_7}{1-D_{I9}D_{I5}}-\frac{S_2S_8}{1-D_{I9}D_{I15}}-\frac{S_3S_5}{1-D_{I2}D_{I4}}+\frac{S_3S_6}{1-D_{I2}D_{I14}}$$
$$-\frac{S_3S_7}{1-D_{I2}D_{I5}}+\frac{S_3S_8}{1-D_{I2}D_{I15}}+\frac{S_4S_5}{1-D_{I10}D_{I4}}-\frac{S_4S_6}{1-D_{I10}D_{I14}}+\frac{S_4S_7}{1-D_{I10}D_{I5}}-\frac{S_4S_8}{1-D_{I10}D_{I15}}\Big) \quad (92)$$

where for a=1, 2,4,5 ,9,10,14,15 b=1,2,9,10 and d=4,5,14,15 $R_{Ia}, S_{Ib,d}$ are given by (27).

Case (iii):The distributions of optional and the mandatory thresholds possess SCBZ property.

If $g(t) = g_{x(1)}(t), f(t) = f_{u(1)}(t)$

$$\begin{aligned} E(T) = {} & (R_1+R_2)C_{I1} - (R_3+R_4)C_{I3} - (R_5+R_6)C_{I6} + (R_7+R_8)C_{I2} + p\big((R_9+R_{10})C_{I9} + (R_{13}+R_{14})C_{I10} \\ & - (R_{11}+R_{12})C_{I11} - (R_{15}+R_{16})C_{I14} - (R_1+R_2)\big((R_9+R_{10})H_{I1,9} + (R_{13}+R_{14})H_{I1,10} - (R_{11}+R_{12})H_{I1,11} \\ & - (R_{15}+R_{16})H_{I1,14}\big) - (R_7+R_8)\big((R_9+R_{10})H_{I2,9} + (R_{13}+R_{14})H_{I2,10} - (R_{11}+R_{12})H_{I2,11} - (R_{15}+R_{16})H_{I2,14}\big) \\ & - (R_3+R_4)\big((R_9+R_{10})H_{I3,9} + (R_{13}+R_{14})H_{I3,10} - (R_{11}+R_{12})H_{I3,11} - (R_{15}+R_{16})H_{I3,14}\big) + (R_5+R_6)\big((R_9+R_{10})H_{I6,9} \\ & + (R_{13}+R_{14})H_{I6,10} - (R_{11}+R_{12})H_{I6,11} - (R_{15}+R_{16})H_{I6,14}\big)\big) \end{aligned} \tag{93}$$

$$\begin{aligned} E(T^2) = 2\big(& (R_1+R_2)C_{I1}^2 - (R_3+R_4)C_{I3}^2 - (R_5+R_6)C_{I6}^2 + (R_7+R_8)C_{I2}^2 + p\big((R_9+R_{10})C_{I9}^2 \\ & + (R_{13}+R_{14})C_{I10}^2 - (R_{11}+R_{12})C_{I11}^2 - (R_{15}+R_{16})C_{I14}^2 - (R_1+R_2)\big((R_9+R_{10})H_{I1,9}^2 \\ & + (R_{13}+R_{14})H_{I1,10}^2 - (R_{11}+R_{12})H_{I1,11}^2 - (R_{15}+R_{16})H_{I1,14}^2\big) - (R_7+R_8)\big((R_9+R_{10})H_{I2,9}^2 \\ & + (R_{13}+R_{14})H_{I2,10}^2 - (R_{11}+R_{12})H_{I2,11}^2 - (R_{15}+R_{16})H_{I2,14}^2\big) - (R_3+R_4)\big((R_9+R_{10})H_{I3,9}^2 \\ & + (R_{13}+R_{14})H_{I3,10}^2 - (R_{11}+R_{12})H_{I3,11}^2 - (R_{15}+R_{16})H_{I3,14}^2\big) + (R_5+R_6)\big((R_9+R_{10})H_{I6,9}^2 \\ & + (R_{13}+R_{14})H_{I6,10}^2 - (R_{11}+R_{12})H_{I6,11}^2 - (R_{15}+R_{16})H_{I6,14}^2\big)\big)\big) \end{aligned} \tag{94}$$

where for a=1, 2, 3,6,9,10,11,14 b=1, 2,3,6and d=9,10,11,14. $C_{Ia}, H_{Ib,d}$ are given by (39).

$$\begin{aligned} & R_1 = \frac{(\delta_1+\mu_1)p_1p_2}{(\delta_1-\delta_2+\mu_1-\mu_2)}, R_2 = \frac{\eta_1q_1p_2}{(\eta_1-\delta_2-\mu_2)}, R_3 = \frac{(\delta_2+\mu_2)p_1p_2}{(\delta_1-\delta_2+\mu_1-\mu_2)}, R_4 = \frac{\eta_2p_1q_2}{(\delta_1-\eta_2+\mu_1)} R_5 = \frac{(\delta_2+\mu_2)q_1p_2}{(\eta_1-\delta_2-\mu_2)}, R_6 = \frac{\eta_2q_1q_2}{(\eta_1-\eta_2)} \\ & , R_7 = \frac{(\delta_1+\mu_1)p_1q_2}{(\delta_1+\mu_1-\eta_2)}, R_8 = \frac{\eta_1q_1q_2}{(\eta_1-\eta_2)}, R_9 = \frac{(\delta_3+\mu_3)p_3p_4}{(\delta_3-\delta_4+\mu_3-\mu_4)} R_{10} = \frac{\eta_3q_3p_4}{(\eta_3-\delta_4-\mu_4)}, R_{11} = \frac{(\delta_4+\mu_4)p_3p_4}{(\delta_3-\delta_4+\mu_3-\mu_4)}, R_{12} = \frac{\eta_4p_3q_4}{(\delta_3-\eta_4+\mu_3)} \\ & R_{13} = \frac{(\delta_3+\mu_3)q_4p_3}{(\delta_3-\eta_4+\mu_3)} R_{14} = \frac{\eta_3q_3q_4}{(\eta_3-\eta_4)}, R_{15} = \frac{(\delta_4+\mu_4)p_4q_3}{(\eta_3-\delta_4-\mu_4)}, R_{16} = \frac{\eta_4q_3q_4}{(\eta_3-\eta_4)} \end{aligned} \tag{95}$$

If $g(t) = g_{x(k)}(t), f(t) = f_{u(1)}(t)$

$$\begin{aligned} E(T) = {} & (R_1+R_2)L_{K1} - (R_3+R_4)L_{K3} - (R_5+R_6)L_{K6} + (R_7+R_8)L_{K2} + p\big((R_9+R_{10})L_{K9} + (R_{13}+R_{14})L_{K10} \\ & - (R_{11}+R_{12})L_{K11} - (R_{15}+R_{16})L_{K14} - (R_1+R_2)\big((R_9+R_{10})M_{K1,9} + (R_{13}+R_{14})M_{K1,10} - (R_{11}+R_{12})M_{K1,11} \\ & - (R_{15}+R_{16})M_{K1,14}\big) - (R_7+R_8)\big((R_9+R_{10})M_{K2,9} + (R_{13}+R_{14})M_{K2,10} - (R_{11}+R_{12})M_{K2,11} - (R_{15}+R_{16})M_{K2,14}\big) \\ & - (R_3+R_4)\big((R_9+R_{10})M_{K3,9} + (R_{13}+R_{14})M_{K3,10} - (R_{11}+R_{12})M_{K3,11} - (R_{15}+R_{16})M_{K3,14}\big) \\ & + (R_5+R_6)\big((R_9+R_{10})M_{K6,9} + (R_{13}+R_{14})M_{K6,10} - (R_{11}+R_{12})M_{K6,11} - (R_{15}+R_{16})M_{K6,14}\big)\big) \end{aligned} \tag{96}$$

$$\begin{aligned} E(T^2) = 2\big(& (R_1+R_2)L_{K1}^2 - (R_3+R_4)L_{K3}^2 - (R_5+R_6)L_{K6}^2 + (R_7+R_8)L_{K2}^2 + p\big((R_9+R_{10})L_{K9}^2 + (R_{13}+R_{14})L_{K10}^2 - (R_{11}+R_{12})L_{K11}^2 \\ & - (R_{15}+R_{16})L_{K14}^2 - (R_1+R_2)\big((R_9+R_{10})M_{K1,9}^2 + (R_{13}+R_{14})M_{K1,10}^2 - (R_{11}+R_{12})M_{K1,11}^2 - (R_{15}+R_{16})M_{K1,14}^2\big) \\ & - (R_7+R_8)\big((R_9+R_{10})M_{K2,9}^2 + (R_{13}+R_{14})M_{K2,10}^2 - (R_{11}+R_{12})M_{K2,11}^2 - (R_{15}+R_{16})M_{K2,14}^2\big) \\ & - (R_3+R_4)\big((R_9+R_{10})M_{K3,9}^2 + (R_{13}+R_{14})M_{K3,10}^2 - (R_{11}+R_{12})M_{K3,11}^2 - (R_{15}+R_{16})M_{K3,14}^2\big) \\ & + (R_5+R_6)\big((R_9+R_{10})M_{K6,9}^2 + (R_{13}+R_{14})M_{K6,10}^2 - (R_{11}+R_{12})M_{K6,11}^2 - (R_{15}+R_{16})M_{K6,14}^2\big)\big)\big) \end{aligned} \tag{97}$$

where for a=1, 2, 3,6,9,10,11,14 b=1, 2,3,6and d=9,10,11,14 $L_{Ka}, M_{Kb,d}$ are given by (44).

If $g(t) = g_{x(k)}(t), f(t) = f_{u(k)}(t)$

$$\begin{aligned} E(T) = {} & (R_1+R_2)P_{K1} - (R_3+R_4)P_{K3} - (R_5+R_6)P_{K6} + (R_7+R_8)P_{K2} + p\big((R_9+R_{10})P_{K9} + (R_{13}+R_{14})P_{K10} \\ & - (R_{11}+R_{12})P_{K11} - (R_{15}+R_{16})P_{K14} - (R_1+R_2)\big((R_9+R_{10})Q_{K1,9} + (R_{13}+R_{14})Q_{K1,10} - (R_{11}+R_{12})Q_{K1,11} \\ & - (R_{15}+R_{16})Q_{K1,14}\big) - (R_7+R_8)\big((R_9+R_{10})Q_{K2,9} + (R_{13}+R_{14})Q_{K2,10} - (R_{11}+R_{12})Q_{K2,11} - (R_{15}+R_{16})Q_{K2,14}\big) \\ & - (R_3+R_4)\big((R_9+R_{10})Q_{K3,9} + (R_{13}+R_{14})Q_{K3,10} - (R_{11}+R_{12})Q_{K3,11} - (R_{15}+R_{16})Q_{K3,14}\big) \\ & + (R_5+R_6)\big((R_9+R_{10})Q_{K6,9} + (R_{13}+R_{14})Q_{K6,10} - (R_{11}+R_{12})Q_{K6,11} - (R_{15}+R_{16})Q_{K6,14}\big)\big) \end{aligned} \tag{98}$$

$$\begin{aligned} E(T^2) = 2\big(& (R_1+R_2)P_{K1}^2 - (R_3+R_4)P_{K3}^2 - (R_5+R_6)P_{K6}^2 + (R_7+R_8)P_{K2}^2 + p\big((R_9+R_{10})P_{K9}^2 + (R_{13}+R_{14})P_{K10}^2 \\ & - (R_{11}+R_{12})P_{K11}^2 - (R_{15}+R_{16})P_{K14}^2 - (R_1+R_2)\big((R_9+R_{10})Q_{K1,9}^2 + (R_{13}+R_{14})Q_{K1,10}^2 - (R_{11}+R_{12})Q_{K1,11}^2 \\ & - (R_{15}+R_{16})Q_{K1,14}^2\big) - (R_7+R_8)\big((R_9+R_{10})Q_{K2,9}^2 + (R_{13}+R_{14})Q_{K2,10}^2 - (R_{11}+R_{12})Q_{K2,11}^2 - (R_{15}+R_{16})Q_{K2,14}^2\big) \\ & - (R_3+R_4)\big((R_9+R_{10})Q_{K3,9}^2 + (R_{13}+R_{14})Q_{K3,10}^2 - (R_{11}+R_{12})Q_{K3,11}^2 - (R_{15}+R_{16})Q_{K3,14}^2\big) + (R_5+R_6)\big((R_9+R_{10})Q_{K6,9}^2 \\ & + (R_{13}+R_{14})Q_{K6,10}^2 - (R_{11}+R_{12})Q_{K6,11}^2 - (R_{15}+R_{16})Q_{K6,14}^2\big)\big)\big) - \frac{1}{\lambda^2}\left(N^2 - M\left(\frac{(R_1+R_2)}{1-B_{I1}} - \frac{(R_3+R_4)}{1-B_{I3}} - \frac{(R_5+R_6)}{1-B_{I6}} + \frac{(R_7+R_8)}{1-B_{I2}}\right)\right. \\ & - \frac{p}{\lambda^2}\left(N^2 - M\left(\frac{(R_9+R_{10})}{1-B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I14}} - (R_1+R_2)\left(\frac{(R_9+R_{10})}{1-B_{I1}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I1}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I1}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I1}B_{I14}}\right)\right.\right. \\ & - (R_7+R_8)\left(\frac{(R_9+R_{10})}{1-B_{I2}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I2}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I2}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I2}B_{I14}}\right) - (R_3+R_4)\left(\frac{(R_9+R_{10})}{1-B_{I3}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I3}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I3}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I3}B_{I14}}\right) \\ & \left.\left. + (R_5+R_6)\left(\frac{(R_9+R_{10})}{1-B_{I6}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I6}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I6}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I6}B_{I14}}\right)\right)\right) \end{aligned} \tag{99}$$

where for a=1, 2, 3,6,9,10,11,14 b=1, 2,3,6and d=9,10,11,14 $P_{Ka}, Q_{Kb,d}$ are given by (48).

If $g(t) = g_{x(1)}(t), f(t) = f_{u(k)}(t)$

$$\begin{aligned}E(T) = {} & (R_1+R_2)R_{I1} - (R_3+R_4)R_{I3} - (R_5+R_6)R_{I6} + (R_7+R_8)R_{I2} + p\big((R_9+R_{10})R_{I9} + (R_{13}+R_{14})R_{I10} - (R_{11}+R_{12})R_{I11} - (R_{15}+R_{16})R_{I14} \\ & - (R_1+R_2)\big((R_9+R_{10})S_{I1,9} + (R_{13}+R_{14})S_{I1,10} - (R_{11}+R_{12})S_{I1,11} - (R_{15}+R_{16})S_{I1,14}\big) - (R_7+R_8)\big((R_9+R_{10})S_{I2,9} + (R_{13}+R_{14})S_{I2,10} \\ & - (R_{11}+R_{12})S_{I2,11} - (R_{15}+R_{16})S_{I2,14}\big) - (R_3+R_4)\big((R_9+R_{10})S_{I3,9} + (R_{13}+R_{14})S_{I3,10} - (R_{11}+R_{12})S_{I3,11} - (R_{15}+R_{16})S_{I3,14}\big) \\ & + (R_5+R_6)\big((R_9+R_{10})S_{I6,9} + (R_{13}+R_{14})S_{I6,10} - (R_{11}+R_{12})S_{I6,11} - (R_{15}+R_{16})S_{I6,14}\big)\big)\end{aligned} \quad (100)$$

$$\begin{aligned}E(T^2) = {} & 2\Big((R_1+R_2){R_{I1}}^2 - (R_3+R_4){R_{I3}}^2 - (R_5+R_6){R_{I6}}^2 + (R_7+R_8){R_{I2}}^2 + p\Big((R_9+R_{10}){R_{I9}}^2 + (R_{13}+R_{14}){R_{I10}}^2 - (R_{11}+R_{12}){R_{I11}}^2 \\ & - (R_{15}+R_{16}){R_{I14}}^2 - (R_1+R_2)\big((R_9+R_{10}){S_{I1,9}}^2 + (R_{13}+R_{14}){S_{I1,10}}^2 - (R_{11}+R_{12}){S_{I1,11}}^2 - (R_{15}+R_{16}){S_{I1,14}}^2\big) - (R_7+R_8)\big((R_9+R_{10}){S_{I2,9}}^2 \\ & + (R_{13}+R_{14}){S_{I2,10}}^2 - (R_{11}+R_{12}){S_{I2,11}}^2 - (R_{15}+R_{16}){S_{I2,14}}^2\big) - (R_3+R_4)\big((R_9+R_{10}){S_{I3,9}}^2 + (R_{13}+R_{14}){S_{I3,10}}^2 - (R_{11}+R_{12}){S_{I3,11}}^2 \\ & - (R_{15}+R_{16}){S_{I3,14}}^2\big) + (R_5+R_6)\big((R_9+R_{10}){S_{I6,9}}^2 + (R_{13}+R_{14}){S_{I6,10}}^2 - (R_{11}+R_{12}){S_{I6,11}}^2 - (R_{15}+R_{16}){S_{I6,14}}^2\big)\Big)\Big) - \frac{1}{\lambda^2}(N^2 - M)\Big(\frac{(R_1+R_2)}{1-B_{I1}} \\ & - \frac{(R_3+R_4)}{1-B_{I3}} - \frac{(R_5+R_6)}{1-B_{I6}} + \frac{(R_7+R_8)}{1-B_{I2}}\Big) - \frac{p}{\lambda^2}(N^2 - M)\Big(\frac{(R_9+R_{10})}{1-B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I14}} - (R_1+R_2)\Big(\frac{(R_9+R_{10})}{1-B_{I1}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I1}B_{I10}} \\ & - \frac{(R_{11}+R_{12})}{1-B_{I1}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I1}B_{I14}}\Big) - (R_7+R_8)\Big(\frac{(R_9+R_{10})}{1-B_{I2}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I2}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I2}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I2}B_{I14}}\Big) - (R_3+R_4)\Big(\frac{(R_9+R_{10})}{1-B_{I3}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I3}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I3}B_{I11}} \\ & - \frac{(R_{15}+R_{16})}{1-B_{I3}B_{I14}}\Big) + (R_5+R_6)\Big(\frac{(R_9+R_{10})}{1-B_{I6}B_{I9}} + \frac{(R_{13}+R_{14})}{1-B_{I6}B_{I10}} - \frac{(R_{11}+R_{12})}{1-B_{I6}B_{I11}} - \frac{(R_{15}+R_{16})}{1-B_{I6}B_{I14}}\Big)\Big)\end{aligned} \quad (101)$$

where for a=1, 2, 3,6,9,10,11,14 b=1, 2,3,6and d=9,10,11,14 $R_{Ia}, S_{Ib,d}$ are given by (51).

V. NUMERICAL ILLUSTRATIONS

The mean and variance of the time to recruitment for the above models are given in the following tables for the cases(i),(ii),(iii) respectively by keeping $\theta_1 = 0.4, \theta_2 = 0.6, \alpha_1 = 0.5, \alpha_2 = 0.8, p = 0.8.$ $\delta_1 = 0.6, \eta_1 = 0.3, \mu_1 = 0.7, \delta_2 = 0.4, \eta_2 = 0.7, \mu_2 = 0.4, \delta_3 = 0.8, \eta_3 = 0.9, \mu_3 = 0.5, \delta_4 = 1, \eta_4 = 1.5, \mu_4 = 0.2, \lambda = 1$ fixed and varying c, k one at a time and the results are tabulated below .

Table – I (Effect of c, k, □ on performance measures)

MODEL-I			c	1.5	1.5	1.5	0.5	1	1.5
			k	1	2	3	2	2	2
			λ	1	1	1	1	1	1
Case (i)	r=1	n=1	E	6.9679	6.3557	6.1484	2.5225	4.4423	6.3557
			V	25.6665	19.3505	17.3979	3.6283	9.9659	19.3505
		n=k	E	6.9679	2.3650	1.3080	1.0798	1.7248	2.3650
			V	25.6665	3.1717	1.0148	0.8753	1.8559	3.1717
	r=k	n=1	E	6.9679	19.0671	33.8154	7.5676	13.3270	19.0671
			V	25.6665	161.4276	488.6549	28.1272	81.0386	161.4276
		n=k	E	6.9679	7.0959	7.1940	3.2394	5.1745	7.0959
			V	25.6665	24.4003	24.2223	6.6552	13.9834	24.4003
Case(ii)	r=1	n=1	E	7.7324	7.0834	6.8728	2.7970	4.9371	7.0834
			V	43.6104	36.0639	33.5832	5.7270	17.6908	36.0639
		n=k	E	7.7324	3.5027	1.4977	1.2753	2.1966	3.5027
			V	43.6104	3.1907	1.4998	1.0853	2.3381	3.1907
	r=k	n=1	E	7.7324	21.2503	37.7996	8.3909	14.8114	21.2503
			V	43.6104	319.2949	998.6626	50.7338	156.1554	319.2949
		n=k	E	7.7324	8.0163	8.2373	3.7368	5.8959	8.0163
			V	43.6104	38.4822	35.3702	7.2110	19.4178	38.4822
Case(iii)	r=1	n=1	E	6.7761	6.1834	5.9831	2.4527	4.3208	6.1834
			V	29.2323	23.1644	21.2936	3.9963	11.6062	23.1644
		n=k	E	6.7761	2.2433	1.2782	1.0315	1.6384	2.2433
			V	29.2323	3.6124	1.1285	0.8973	2.0285	3.6124
	r=k	n=1	E	6.7761	18.5503	32.9067	7.3582	12.9625	18.5503
			V	29.2323	196.1130	608.2081	31.0609	87.1721	196.1130
		n=k	E	6.7761	6.7298	7.0299	3.0944	4.9151	6.7298
			V	29.2323	28.0247	26.4670	6.0124	14.9796	28.0247

MODEL-II			c	1.5	1.5	1.5	0.5	1	1.5
			k	1	2	3	2	2	2
			λ	1	1	1	1	1	1
Case (i)	r=1	n=1	E	3.0491	2.4841	2.2929	1.1966	1.8442	2.4841
			V	7.1810	4.4459	3.6707	1.1672	2.5502	4.4459
		n=k	E	3.0491	1.0669	0.6046	0.6467	0.8546	1.0669
			V	7.1810	0.9564	0.3209	0.4026	0.6514	0.9564
	r=k	n=1	E	3.0441	7.4522	12.6107	3.5897	5.5326	7.4522
			V	7.1810	40.0132	111.0337	10.5046	22.9520	40.0132
		n=k	E	3.0441	3.2008	3.3253	1.9402	2.5638	3.2008
			V	7.1810	6.4733	6.0806	2.3297	4.1530	6.4733
Case(ii)	r=1	n=1	E	4.4497	3.8440	3.6369	1.6790	2.7663	3.8440
			V	10.6673	6.6718	5.4893	1.7016	3.8162	6.6718
		n=k	E	4.4497	1.5259	0.8502	0.7959	1.1631	1.5259
			V	10.6673	1.4048	0.4678	0.5362	0.9324	1.4048
	r=k	n=1	E	4.4497	11.5320	20.0024	5.0370	8.2990	11.5320
			V	10.6673	52.3579	144.2232	11.9566	28.8134	52.3579
		n=k	E	4.4497	4.5776	4.6762	2.3877	3.4894	4.5776
			V	10.6673	9.5917	9.0489	3.2339	6.0657	9.5917
Case(iii)	r=1	n=1	E	2.9380	2.3862	2.1980	1.1588	1.7761	2.3862
			V	7.1321	4.5063	3.7663	1.1488	2.5532	4.5063
		n=k	E	2.9380	1.0271	0.5761	0.6355	0.8291	1.0271
			V	7.1321	0.9467	0.3126	0.3945	0.6393	0.9467
	r=k	n=1	E	2.9380	7.1587	12.0889	3.4765	5.3284	7.1587
			V	7.1321	33.4547	94.9477	6.4008	17.4633	33.4547
		n=k	E	2.9380	1.0271	0.5761	0.6355	0.8291	1.0271
			V	7.1321	0.9467	0.3126	0.3945	0.6393	0.9467

MODEL-III			c	1.5	1.5	1.5	0.5	1	1.5
			k	1	2	3	2	2	2
			λ	1	1	1	1	1	1
Case (i)	r=1	n=1	E	8.7025	8.0800	7.8698	3.1046	5.5952	8.0800
			V	34.8211	26.8975	24.4103	4.8250	13.6770	26.8975
		n=k	E	8.7025	2.9422	1.6226	1.2767	2.118	2.9422
			V	34.8211	4.2573	1.3523	1.0932	2.4337	4.2573
	r=k	n=1	E	8.7025	24.2401	43.2829	9.3139	16.7855	24.2401
			V	34.8211	225.9177	691.1652	37.2158	111.9030	225.9177
		n=k	E	8.7025	8.8266	8.9241	3.83	6.3355	8.8266
			V	34.8211	32.4314	31.1711	7.2859	17.6793	32.4314
Case(ii)	r=1	n=1	E	11.9437	11.2767	11.0507	4.1989	7.7414	11.2767
			V	44.2200	33.4379	30.0115	6.1929	17.2159	33.4379
		n=k	E	11.9437	4.0179	2.2079	1.6500	2.8369	4.0179
			V	44.2200	5.4334	1.7299	1.4021	3.1300	5.4334
	r=k	n=1	E	11.9437	33.8301	60.7777	12.5968	23.2243	33.8301
			V	44.2200	278.3881	841.5110	47.3386	139.4601	278.3881
		n=k	E	11.9437	12.0536	12.1431	4.95	8.5106	12.0536
			V	44.2200	40.8646	39.0814	9.3189	22.4964	40.8646
Case(iii)	r=1	n=1	E	8.4755	7.8757	7.6735	3.0222	5.4512	7.8757
			V	40.0820	32.4776	30.0962	5.3736	16.0863	32.4776
		n=k	E	8.4755	2.8689	1.5833	1.2449	2.0589	2.8689
			V	40.0820	4.8388	1.5246	1.1397	2.6751	4.8388
	r=k	n=1	E	8.4755	23.6271	42.2037	9.0665	16.3536	23.6271
			V	40.0820	276.5466	864.3347	42.3183	133.8743	276.5466
		n=k	E	8.4755	8.6067	8.7081	3.7341	6.1768	8.6067

			V	40.0820	37.8112	36.6164	7.7674	19.9579	37.8112

VI. FINDINGS

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment for all the models are reported below.

i. It is observed that the mean time to recruitment decreases, with increase in k for the cases r=1,n=1 and r=1 ,n=k but increases for the cases r=k,n=1 and r=k,n=k.
ii. If c increases, the average number of exits increases, which, in turn, implies that mean and variance of the time to recruitment increase for all the models.

VII. CONCLUSION

Note that while the time to recruitment is postponed in model-III, the time to recruitment is advanced in model-I and II .Therefore from the organization point of view, model III is more preferable.

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